# COMPTES RENDUS
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## DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

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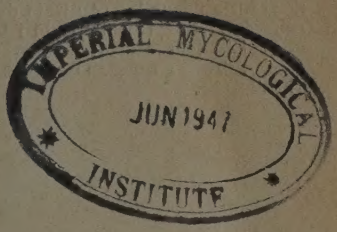


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*MATHEMATICS*

# ON AN APPLICATION OF THE THEORY OF HERMITIAN OPERATORS TO THE GENERALIZED PROBLEM OF MOMENTS

By M. S. LIVSHITZ

*(Communicated by S. N. Bernstein, Member of the Academy, 20. XI. 1943)*

In the present note we shall prove a theorem on Hermitian operators and deduce from it certain conditions under which a «generalized problem of moments» admits of a solution. A simple application of this result shall lead us to a proposition which contains the theorems of S. Bernstein, S. Bochner and M. Krein on the representation of exponentially convex and Hermite-positive functions by Stieltjes integrals.

1. Let $\mathfrak{H}$ be a Hilbert space, and $A$ a closed Hermitian operator with the domain of definition $D_A$ which is dense in $\mathfrak{H}$.

As is well known ([1,3]), an one-parametric family $E_t$ $(-\infty < t < \infty)$ of bounded self-adjoint operators is called the spectral function of the operator $A$ if for every $f \in \mathfrak{H}$*:

1°. $(E_t f, f)$ is non-decreasing for increasing $t$;

2°. $E_t f$ is a function of $t$ continuous from the left;

3°. $E_t f \to 0$ for $t \to -\infty$ and $E_t f \to f$ for $t \to +\infty$;

4°. for every finite interval $\Delta$ and $f \in \mathfrak{H}$ the element $E_\Delta f$ belongs to the domain of definition of the conjugated operator $A^*$ and $A^* E_\Delta f = \int_\Delta t\, dE_t f$.

Theorem 1. *If to the elements $f \in \mathfrak{H}$ and $u \in \mathfrak{H}$ can be made to correspond a continuous number function $f(x)$ $(-\infty < x < \infty)$ and a vector function $g_x \in D_A$ $(-\infty < x < \infty)$ in such a way that*

$$f - f(x)\, u = (A - xE)\, g_x \quad (-\infty < x < \infty) \tag{1}$$

*then*

$$f = \int_{-\infty}^{\infty} f(t)\, dE_t u, \quad (f, f) = \int_{-\infty}^{\infty} |f(t)|^2\, d(E_t u, u), \tag{2}$$

*where $E_t$ is any spectral function of the operator $A$.*

Proof. First suppose that $A$ is a self-adjoint operator. Then it will have a unique spectral function $E_t$ $(-\infty < t < \infty)$ which will be orthogonal.

* Another, more natural definition of the spectral function based on the investigation of M. Neumark ([5]), was recently proposed by M. Krein ([2]).

From (1) follows

$$E_\Delta f - f(x) E_\Delta u = \int_\Delta (t-x)\, dE_t g_x$$

Hence, if the interval $\Delta$ contains the point $x$

$$\| E_\Delta f - f(x) E_\Delta u \| \leqslant \Delta \| E_\Delta g_x \| \tag{3}$$

Let now $x_0$ belong to the point spectrum of the operator $A$ and $E_\Delta = E_{x_0+0} - E_{x_0}$. Then from (3) we obtain

$$E_\Delta f = f(x_0) E_\Delta u$$

Hence without loss of generality we may assume that $A$ has only a continuous spectrum. Under this supposition from (3) and the inequality

$$\| f(x) E_\Delta u - \int_\Delta f(t)\, dE_t u \| \leqslant \varepsilon_\Delta (E_\Delta u, u)$$

where $\varepsilon_\Delta$ is the oscillation of the function $f(x)$ in the interval $\Delta$, it follows that

$$\lim_{\Delta \to 0} \frac{E_\Delta f - \int_\Delta f(t)\, dE_t u}{\Delta + (E_\Delta u, u)} = 0$$

Hence we conclude that for any interval $\Delta$

$$E_\Delta f = \int_\Delta f(t)\, dE_t u$$

which proves the first of the equalities (2). The second of these equalities follows in this case from the first in virtue of the orthogonality of the spectral function $E_t$.

For the proof of the theorem in the general case we apply a theorem of M. Neumark (3), according to which for every spectral function $E_t$ may be constructed a self-adjoint operator $A'$ in a certain wider Hilbert space $\mathfrak{H}' \supset \mathfrak{H}$ so that

$$A'f = Af \quad (f \in D_A) \quad \text{and} \quad E_t = E_1 E'_t \quad (-\infty < t < \infty)$$

where $E'_t$ is the spectral function of the operator $A'$, and $E_1$, the projection operator of $\mathfrak{H}'$ on $\mathfrak{H}$.

Then we shall have by what has been already proved

$$f = \int_{-\infty}^{\infty} f(t)\, dE'_t u, \qquad (f, f) = \int_{-\infty}^{\infty} |f(t)|^2\, d(E'_t u, u)$$

whence

$$f = E_1 f = \int_{-\infty}^{\infty} f(t)\, dE_1 E'_t u = \int_{-\infty}^{\infty} f(t)\, dE_t u$$

and since

$$(E'_t u, u) = (E'_t u, E_1 u) = (E_1 E_t u, u) = (E_t u, u) \quad (-\infty < t < \infty)$$

the theorem is completely proved.

2. Let $G$ be a certain linear aggregate of functions $f(t) (-\infty < t < \infty)$ assuming, in general, complex values. By $G\bar{G}$ we denote the linear envelope of the aggregate of functions $F(t)$ $(-\infty < t < \infty)$ of the form $F(t) = f(t)\overline{g(t)}$ where $f \in G$ and $g \in G$.

The linear functional $P$, defined on $G\bar{G}$, shall be called Hermite-positive, if it satisfies the condition

$$P(f\bar{f}) \geqslant 0 \quad (f \in G)$$

It is easily seen that then

$$P(f\bar{g})=\overline{P(g\bar{f})} \quad (g,f\in G)$$

The functional $P$ defined on $G\bar{G}$ will be evidently Hermite-positive, if it admits of the representation

$$P(f\bar{g})=\int_{-\infty}^{\infty} f(t)\,\overline{g(t)}\,d\sigma(t) \quad (g,f\in G) \tag{4}$$

where $\sigma(t)(-\infty<t<\infty)$ is a certain bounded non-decreasing function.

Under the «generalized problem of moments» we understand the determination of conditions under which, conversely, from the Hermite-positiveness of the functional $P$ follows the possibility of its representation in the form (4), as well as the determination of all solutions $\sigma$. For the solution of this question we establish Theorem 2.

Theorem 2. *Let the linear aggregate $G$ of functions $f(t)$ $(-\infty<f<\infty)$ and the Hermite-positive functional $P$ on $G\bar{G}$ satisfy the following two conditions:*

1°. $1\in G$

2°. *The set $D$ of functions $\varphi\in G$ for which $t\varphi\in G$, $P$ is dense in $G$ in the sense of $P$-norm*

$$\|f\|_P=\sqrt{P(f\bar{f})} \quad (f\in G)$$

*Then, $f(t)$ being a certain continuous function, the function*

3°. $\frac{f(t)-f(x)}{t-x}\in G \quad (-\infty<x<\infty)$ (5)

*if considered as a function of $t$,*

$$P(f\bar{f})=\int_{-\infty}^{\infty}|f(t)|^2\,d\sigma(t) \tag{6}$$

*where $\sigma(t)$ $(-\infty<t<\infty)$ is a bounded non-decreasing function not depending on the choice of the function $f$.*

*If any function $f\in G$ is continuous and satisfies the condition 3°, then for the same function $\sigma$ holds the representation (4) of the functional $P$.*

Proof. Define in $G$ the scalar product by setting

$$(f,g)=P(f\bar{g})$$

Identifying in $G$ any two elements $g\in G$ and $f\in G$ for which

$$\|g-f\|=(g-f,g-f)^{1/2}=0$$

take the closure of $G$ up to a complete Hilbert space $\mathfrak{H}$.

In $\mathfrak{H}$ define the operator $A_1$ by putting $A_1\varphi=t\varphi$ $(\varphi\in D)$. We prove the uniqueness of the operator $A_1$; to this end we have to prove that if $\|\varphi_0\|=0$ $(\varphi_0\in D)$, then $\|t\varphi_0\|=0$. In virtue of Schwarz' inequality: $|(\varphi_0,f)|\leqslant\|\varphi_0\|\,\|f\|$ we shall have $(\varphi_0,f)=0$ $(f\in G)$ and, consequently

$$(t\varphi_0,\varphi)=P(t\varphi_0,\bar{\varphi})=(\varphi_0,t\varphi)=0 \quad (\varphi\in D)$$

and since $D$ is dense in $G$, hence follows $(t\varphi_0,t\varphi_0)=0$.

In virtue of the fact that $(A_1\varphi,\psi)=(\varphi,A_1\psi)$ $(\varphi,\psi\in D)$, and that the domain of definition $D$ of the operator $A_1$ is by assumption dense in $G$ and so also in $\mathfrak{H}$, the operator $A_1$ admits of a closed Hermitian extension $A$.

Let now the function $f(t)\in G$ satisfy the condition (5); then

$$f(t)-f(x)1=(t-x)\frac{f(t)-f(x)}{t-x}$$

or

$$f-f(x)1=(A-xE)g_x,\quad g_x=\frac{f(t)-f(x)}{t-x}$$

Choosing an arbitrary spectral function $E_t$ of the operator $A$ and applying Theorem 1, we obtain

$$P(ff)=(f, f)=\int_{-\infty}^{\infty}|f(t)|^2 d(E_t 1, 1)$$

If a certain continuous function $g\in G$ also satisfies (5), then, applying the relation (6) to the functions $f\pm g$, $f\pm ig$, we find that for $\sigma(t)=(E_t 1, 1)$ $(-\infty<t<\infty)$ (3) also holds.

3. We state two examples of the application of Theorem 2.

a) Let $G$ be the set of all polynomials in $t$; the $1\in G$, $G\bar{G}$ and $D$ coincide with $G$. Thus, for any Hermite-positive functional $P$ on $G\bar{G}$ the conditions of Theorem 2 will be satisfied and, consequently, there will exist a bounded non-decreasing function $\sigma(t)$ $(-\infty<t<\infty)$ such that

$$P(t^k)=\int_{-\infty}^{\infty} t^k d\sigma(t) \quad (k=0, 1, 2, \ldots)$$

On the other hand, it is evident that any sequence of numbers $\{s_k\}_0^\infty$ defines on $G$ a linear functional $P$, if we put $P(t^k)=s_k$ $(k=0, 1, 2, \ldots)$, and this functional will by definition be Hermite-positive on $G\bar{G}=G$ if all forms

$$\sum_0^n s_{j+k}\xi_j\bar{\xi}_k=P\left(\left|\sum_0^n \xi_j t^j\right|^2\right)\geqslant 0 \quad (n=0, 1, 2, \ldots)$$

We thus obtained the solution of the power problem of moments.

b) As the second example we obtain the following

Theorem 3. *Let $Q$ denote a certain convex set in the complex plane containing the point $O$ and symmetric with respect to the real axis, and $^1/_2Q$, the set of all points of the form $^1/_2 z$ where $z\in Q$.*

*Then in order that a continuous function $F(z)$ $(z\in {}^1/_2Q)$ be representable as*

$$F(z)=\int_{-\infty}^{\infty} e^{zt} d\sigma(t) \quad (z\in {}^1/_2Q) \tag{7}$$

*where $\sigma(t)$ $(-\infty<t<\infty)$ is a certain bounded non-decreasing function, it is necessary and sufficient that for any $z_1\in {}^1/_2Q, \ldots, z_n\in {}^1/_2Q$ $(n=1, 2, \ldots)$ the forms*

$$\sum_{j, k=1}^n F(z_j+\bar{z}_k)\xi_j\bar{\xi}_k$$

*should be non-negative.*

Proof. The necessity of the condition follows trivially from the representation (7). For the proof of its sufficiency choose for $G$ the aggregate of functions $f(t)$ of the form

$$f(t)=\int_{Q'} e^{t\zeta} d\mu(\zeta) \quad (-\infty<t<\infty)$$

where $Q'$ is any bounded convex set contained in $^1/_2Q$ and $d\mu(\zeta)$ is a certain Stieltjes differential on $Q'$; besides define on $G\bar{G}$ the functional $P$ putting

$$P(f\bar{g}) = \int_{Q_1'} \int_{Q_2'} F(\zeta_1 + \zeta_2)\, d\mu_1(\zeta_1)\, \overline{d\mu_2(\zeta_2)}$$

if

$$f(t) = \int_{Q_1'} e^{t\zeta}\, d\mu_1(\zeta), \qquad g(t) = \int_{Q_2'} e^{t\zeta}\, d\mu_2(\zeta)$$

It is easily seen that the aggregate $G$ and functional $P$ so defined satisfy all conditions of Theorem 2, which enables us to assert the existence of the representation (4). Putting in (4) $f(t) = e^{t\frac{z}{2}}$, $g(t) = e^{t\frac{\bar{z}}{2}}$ $(z \in {}^1/_2 Q)$ and observing that $P(f\bar{g}) = F(z)$, we prove the theorem.

We observe that if $Q$ is the whole imaginary axis, or a segment of it, then our Theorem 3 becomes the theorem of S. Bochner ([4]), or, correspondingly, the theorem or M. Krein ([5],[6]) on Hermite-positive functions; if $Q$ is a real segment, the real half-axis, or the whole real axis, then our Theorem 3 transforms into theorem of S. Bernstein ([7]) on exponentially convex functions.

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*ASTRONOMY*

# ON THE ORIGIN OF VISUAL BINARY STARS AND THE CHARACTER OF THEIR ORBITS

By O. J. SCHMIDT, Member of the Academy

1. No less than a third of the total number of stars are binaries. Within recent years the number of recorded binaries has run into many thousands, and the number of calculated orbits has amounted to several hundreds. Up to the present, however, no elaborate theory of the origin of binary stars has been created. The theory on which binaries are supposed to arise from the capture of stars that formerly existed as independent individuals has not lately been supported by anyone, and the theory locating the formation of double stars in adjacent points of the primary nebula is confirmed neither by computation nor by facts. Even the theory of the formation of double stars by fission which is more widely recognized has failed, at least in the case of visual stars, to explain any of the striking regularities established by observation and statistical evidence.

For visual binary stars, *i. e.* such as can be separated by the eye with the aid of a telescope, the following laws are most typical:

a) Unlike planets the excentricity of whose orbit is near to zero and comets for which it is near to unity, visual binary stars move along orbits which exhibit every possible kind of excentricity. The mean excentricities of orbit in the case of these stars is near to $^1/_2$.

b) On the average, a large excentricity is a typical feature of a large orbit (a large semi-axis $a$), while small excentricities $e$ correspond to small $a$. In other words, the larger the orbit, the more it is elongated, and, accordingly, smaller orbits will be found to deviate less from the circular form.

c) Small angular distances between the components of a binary system are much more often encountered than large ones, and the frequency of occurrence of stars with different angular distances has been expressed in per cent on the basis of statistical data.

No theory can claim to account for the origin of binaries unless it is able to explain these regularities.

It seems that the rotation of the Galaxy which was definitely established in 1926 has not been yet taken into account in building a theory of binaries. This paper has for object to place the theory of capture upon a new foundation by taking account of the Galactic rotation.

Accepting the capture of stars as our working hypothesis, we shall analyse the conditions of this capture, if it takes place at all, and draw from the capture hypothesis a series of conclusions as to the character of the resulting orbits of relative motion of one star of a binary system about the other. Proceeding thus, we shall arrive by a logical way at certain laws from which the above-mentioned regularities observed in visual binaries will follow. We shall find the re-

sults obtained to be of quantitative and not merely of qualitative nature, and they also will coincide perfectly with the data of stellar statistics.

2. We imagine the motion of stars in the Galaxy to proceed under the action of a central force. For the sake of concreteness we assume this force to be Newtonian attraction by a mass M which is conceived as concentrated in the centre of the Galaxy.

To be more concrete still, we assume that the stars move in circles and their orbits lie in different planes passing through the centre. These restrictions are not essential for our conclusions, and their sole purpose is to render our description more definite.

If $v_1$ and $v_2$ are the velocities of two stars at a distance $R$ and $R-d$ from the centre of the Galaxy, respectively, then in the case of circular motion

$$v_1^2 = k^2 \frac{M}{R}, \quad v_2^2 = k^2 \frac{M}{R-d}$$

where $k^2$ is the constant of gravitation. Let $\alpha$ be the angle between the plane of the two orbits. The quantity $d$ is the difference between the distances from the centre, and at the same time it is the shortest possible distance between our stars, which is only attained when both stars simultaneously come near to the nodes of their respective orbits. If the capture is at all possible, it must take place at this time of simultaneous passage of the two stars through the position of shortest distance.

The relative velocity $L$ of the stars at that instant is defined as the geometrical difference between the velocities $v_1$ and $v_2$, *i. e.*

$$L^2 = v_1^2 + v_2^2 - 2v_1 v_2 \cos\alpha.$$

Expanding in a series of powers of $d$ and omitting the terms of order higher than two, we obtain

$$L^2 = \frac{k^2 M}{R} \left\{ \left(2 + \frac{d}{R}\right)(1 - \cos\alpha) + \frac{d^2}{R^2}\left(1 - \frac{3}{4}\cos\alpha\right) \right\}$$

If the angle $\alpha$ is very small, the preponderant term in the brackets is $\frac{d^2}{4R^2}$; if, on the contrary, the angle $\alpha$ is not too small, the term $2(1-\cos\alpha)$ prevails, which is independent of $d$. In order that the angle $\alpha$ be «sufficiently» large, it is necessary, as may be easily seen, that

$$\alpha^2 \gg \frac{d}{2R}.$$

Thus, for $R = 10000$ parsecs (in the case of the Sun) and $d = 400$ astr. units it will suffice that $\alpha \gg 10^{-7}$, or about 0.02 seconds. Thus, the angle $\alpha$ is «sufficiently» large if it measures some small fractions of a second.

In what follows just this case of a sufficiently large angle will be considered. Then the relative velocity will, as has been shown above, be independent of the distance $d$ in first approximation. Later we shall find that the class of stars thus defined coincides with that of visual binaries.

The relative velocity $L$ is at the same time the initial velocity of the relative motion of the one star about the other after capture has taken place. According to the well-known formula of celestial mechanics

$$L^2 = k^2 2M \left(\frac{1}{d} - \frac{1}{2a}\right)$$

where $M$ is the sum of the masses of the two stars, $a$ is the semi-axis major

of the relative orbit. Equating the two expressions of $L$ gives

$$\frac{1}{d}-\frac{1}{2a}=\frac{1}{\rho} \qquad (1)$$

where $\rho = R\frac{M}{M}\frac{1}{2\sin^2\frac{a}{2}}$ does not depend on $d$. It is easy to see that in the shortest distance position $L$ is directed at right angles to $d$. By virtue of this perpendicularity the stars are in the position of periastron or apoastron. If $e$ is the excentricity of the relative orbit, then $d=a(1-e)$ in the first case, and $d=a(1+e)$ in the second. Therefore, in the case of periastron formula (1) can be represented as well in the following two forms:

$$1-\frac{1-e}{2}=\frac{d}{\rho} \qquad (2)$$

$$\frac{1}{1-e}-\frac{1}{2}=\frac{a}{\rho} \qquad (3)$$

In the case of apoastron $e$ changes its sign. From these formulae the meaning of the quantity $\rho$ becomes evident. This quantity is equal to the limiting value of the distance $d$, when the capture leads to a parabolic orbit ($a$ is infinitely large). For a circular orbit we have $d=a=\frac{\rho}{2}$. If $d>\frac{\rho}{2}$, the stars are in the position of periastron, and if $d<\frac{\rho}{2}$, they are in apoastron.

3. Simple as are the formulae (2) and (3), all the above-mentioned statistical laws for visual boundaries can be deduced from them.

All values of $d$ from 0 to $\rho$ are equiprobable, for the density of a stream of stars involved in the motion of the Galaxy may be regarded as uniform within a bounded volume. Now, according to formula (2), the quantities $e$ and $d$ are connected by a linear relation and therefore the values of $e$ ranging from 0 to 1 are equiprobable also and the mean value of $e$ is

$$\bar{e}=1/2$$

which accounts for the first of the aforementioned regularities.

Formula (3) shows that the dependence between $a$ and $e$ is hyperbolic. It may easily be seen that $a$ increases with $e$ (for a given $\rho$), and conversely. In the case of apoastron where $1+e$ stands in the denominator, the relation is reverse. In the celestial statistics the two cases coincide, and to one value of $e$ there correspond two values $a_1$ and $a_2$: one for periastron, the other for apoastron. For their mean we easily obtain from our formulae the relation

$$\frac{\bar{a}}{\rho}=\frac{a_1+a_2}{2}\frac{1}{\rho}=\frac{1}{1-e^2}-\frac{1}{2}$$

which means that on the average $\bar{a}$ also grows with $e$. This explains the second dependence qualitatively.

Proceeding to a quantitative verification of our formulae, it must be realized that the magnitude of $\rho$ is, generally speaking, different for different binaries. Therefore the above formulae cannot be applied right away to a given pair of stars. But in case we have a sufficient accumulation of statistical data at our disposal, we can refer it to a certain average value of $\rho$ and on this assumption analyse the dependence between $a$ and $e$ in the mean.

For the orbits of telescopic binaries the greatest number of cases is comprised by Russell's statistics referring to wide pairs (see his well-known text-

book). 500 pairs with an average period estimated as 2000 years and a mean excentricity of 0.61 come within its scope. It also covers 800 other pairs with an average period of 5000 years and an excentricity of 0.76.

To the period ratio of 5000 : 2000 there corresponds a semi-axis ratio of $\left(\frac{5}{2}\right)^{2/3} = 1.84$. And if we introduce into formula (3) at first $e = 0.76$ and then 0.61, and take the ratio between the left-hand sides of the formula, the result will be 1.79, and it is seen that the two figures coincide within half a unity in the second decimal place.

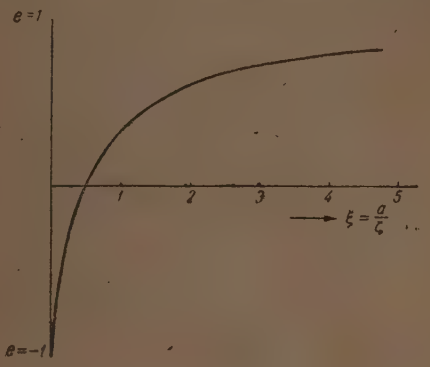

Fig. 1

4. A still more extensive statistical material has been provided by Aitken (1) who in the Lick Observatory has determined all stars of the Northern Hemisphere up to the ninth magnitude having a neighbour at an angular distance of less than 5″. This imposing work which took many years to accomplish it, gives exceptionally reliable statistical data. Among the stars ranging from the first to the ninth magnitude, which number more than 100000, there have proved to be 5400 visual binaries with an angular distance less than 5″.

On the page 264 of his book Aitken presents a table of the distribution of these stars in per cent according to the angular distance

| 0—1″ | 1—2″ | 2—3″ | 3—4″ | 4—5″ |
|---|---|---|---|---|
| 42% | 23% | 15% | 11% | 9% |

Moreover, for the interval from 0 to $\frac{1}{2}$″ a value of 24 per cent follows from the absolute figures of Aitken.

What can tell our theory about the expected distribution?

The curve shown in Fig. 1 brings out the relation between $e$ and $\xi = \frac{a}{\rho}$, according to formula (3). In order to comprise the case of apoastron, we shall assign $e$ values ranging from $-1$ to $+1$.

Since the values of $e$ are, as pointed out above, equiprobable, the probability that the value of $\xi$ is included between some limits $\xi_1$ and $\xi_2$ is determined by the difference between the respective values $e_1$ and $e_2$. Thus, the probability that $\xi$ is included between 0 and $\xi$ is proportional to $1+e$.

From formula (3) we have

$$1 + e = \frac{4\xi}{2\xi + 1} \qquad (4)$$

where $\xi = \frac{a}{\rho}$. We shall take it for granted that the various $\rho$ can be substituted with their mean value $\bar{\rho}$. Let us also take into consideration that the angular distances do not exactly correspond to the semi-axes, but are rather distorted by the perspective and by differences in the position of the star as it moves along its orbit. We can assume, in view of the vast number of objects covered by the statistical data, that the length $a$ is on the average proportional to the angular distance $u''$. Then $\frac{a}{\rho} = \xi = \sigma u''$, where $\sigma$ is the factor of proportionality which in the mean may be taken to be constant.

Since all double stars up to a certain magnitude are not comprised by Aitken's statistics, but only those for which $u''<5''$, it is necessary to add another factor of proportionality $q$, and replace our formula by

$$S\ \%=q(1+e)=q\ \frac{4\sigma u''}{2\sigma u''+1} \qquad (5)$$

Let us determine the horizontal and vertical scale of the curve in such a way as to make $S=100$ per cent correspond to $u''=5''$, and $S=42$ per cent as given by Aitken, to $u''=1''$. From these conditions it is easy to determine

$$\sigma=0.19, \qquad q=76\%$$

so that

$$S=76\ \frac{0.76u''}{0.38u''+1} \qquad (6)$$

In the following table are compared the values of $S$ as given by Aitken in the above-mentioned table with those calculated according to formula (6).

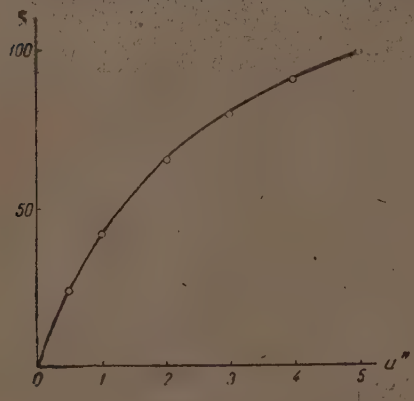

Fig. 2

| Angular distance | Observed | Calculated |
|---|---|---|
| 0—½'' | 24 | 24 |
| 0—1 | 42 | 42 |
| 0—2 | 65 | 66 |
| 0—3 | 80 | 81 |
| 0—4 | 91 | 92 |
| 0—5 | 100 | 100 |

The figures show a striking coincidence in spite of the fact that in determing the scales only two points (the second and the last) have been made to coincide, while the remaining four are in no way affected by our choice.

Fig. 2 represents the theoretical curve drawn according to formula (6) along with points presenting the data of Aitken's table compiled by him from statistical data.

So it appears that the conclusions arrived at by supposing visual binary stars to originate from capture in a rotating Galaxy explain the regularities known for these stars quantitatively as well as qualitatively.

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*PHYSICS*

# SCATTERING OF MESONS IN THE CASE OF STRONG COUPLING

By I. POMERANCHUK

*(Communicated by A. I. Alikhanov, Member of the Academy, 10. XI. 1943)*

As well known, the method of perturbation cannot be applied for the description of interaction between mesons and nucleons. Considering pseudo-scalar mesons Pauli and Dancoff (1) (quoted below as loc. cit.) have used the theory of strong coupling without any application of perturbation calculation. They introduced a new characteristic length $a$ playing the part of dimensions of the source (nucleon), which enters in the formulae for effective cross-section for the scattering of mesons by neutrons or protons, as well as in the expression for anomalous magnetic moment and proper «mesonic» energy of nucleons. Taking this $a$ equal to the Compton wave-length of the proton $\frac{\hbar}{Mc}$, Pauli and Dancoff got satisfactory results for the scattering and magnetic moment. We may remark, however, that with this value of $a$ the absolute value of the proper energy $E$ happens to be much greater than $Mc^2$. From the formula (63) of l. c. we have

$$E = -\frac{3}{4}\frac{g^2}{\varkappa^2}\hbar c I \qquad (1)$$

Further, from (14) of l. c. follows that $I \geqslant \frac{1}{a^2}$

$$|E| \geqslant \frac{3g^2\hbar c}{4\varkappa^2 a^2} \qquad (2)$$

Substituting $\frac{g^2}{\varkappa a} = 7$ (1) and taking the mass of the meson equal to 0.1 of protonic mass, we get with $a = \frac{\hbar}{Mc}$

$$|E| \geqslant \frac{3}{4}\,7 \cdot 10 Mc^2 = 52.5 Mc^2 \qquad (3)$$

This result shows that the choice of $a$ made in loc. cit. met with difficulties which presumably could be removed only by means of more detailed considerations connected with the analysis of proper mass. The question of the proper nucleonic mass would play no part only if (1) would lead to the value of $E$ of the order of $Mc^2$ or less. If, on the contrary, one admits the validity of considerations made in loc. cit., but chooses $a$ satisfying the

requirement that $E$ must be of the order of $Mc^2$, then the effective cross-section for scattering $q = 6\pi a^2$ (formula 104 C of loc. cit.) would be equal to

$$a = \left(\frac{3g^2}{4\varkappa^3}\frac{\hbar}{Mc}\right)^{\frac{1}{3}} \tag{4}$$

$$q = 6\pi\left(\frac{3g^2\hbar}{4\varkappa^2 Mc}\right)^{\frac{2}{3}} \tag{5}$$

The anomalous magnetic moment [loc. cit. (126)] with the new value of $a$ taken from (4) proves equal to

$$M_z = \frac{e}{36\pi a}\frac{g^2}{\varkappa} = \frac{eg^2}{36\varkappa^2}\left(\frac{4\varkappa^3 Mc}{3g^2\hbar}\right)^{\frac{1}{3}} \tag{6}$$

Comparing this latter expression with experimental value of the magnetic moment of the neutron, which was found to be equal to 1.93 of the proton's magnetic moment, one gets for $g$

$$g = 1.4$$

Substituting now (7) in (5), we obtain

$$q = 2 \cdot 10^{-25}\,\text{cm}^2 \tag{7}$$

The available experimental data (2) for observed anomalous scattering of mesons correspond to an effective cross-section which is 1000 times smaller. This situation met with in the theory of strong coupling corresponds rather close to the state of affairs in the habitual theory dealing with the perturbation method, where indeed it is not found possible to bring into agreement the values of $g$ leading both to required strong nuclear forces, on the one hand, and to observed small scattering, on the other. It seems that a satisfactory explanation of the scattering of mesons by nucleons is not possible under the scope of current theory without introducing into consideration the questions connected with the proper mass of nucleon.

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*TECHNICAL PHYSICS*

# DEVELOPMENT OF PHOTOGRAPHIC PHOTOMETRY WITHOUT OBJECTIVES

By G. A. TIKHOV, Corresponding Member of the Academy

In 1942 I published an article on photographic photometry without objectives (1). The principle of this method is that a bright star or a planet gives in a lensless camera a certain illumination on the photographic plate, which results in a blackening on the negative. In the case of a parallel beam the illumination on the photographic plate is naturally independent of the distance between the plate and the diaphragm placed instead of the objective which is removed; the diameter of the circle exposed, moreover, is equal to that of the aperture of the diaphragm.

Observations have shown, however, that in order to avoid troubles caused by diffraction, the diameter of the aperture of the diaphragm in a camera 1.6 m long should not be less than 10 mm. It has been proved by experiments that in order to obtain a sufficient blackening from Jupiter an exposure of 15 minutes is necessary in the photographic area, while in the photovisual area an hour and a half is required; in the case of Saturn the exposure needed is 9 hours and more.

I was able to determine in this way the colour index of the sky relative to the Moon and the colour index of Saturn relative to Jupiter.

Simple as it may seem, and in spite of its photometrical advantages, this method presents considerable technical difficulties, for it requires very long exposures even with bright stars and planets.

Thinking over the possibility of increasing the illumination on the photographic plate and thus reducing the time of exposure, I arrived at the following idea.

Let us suppose that in front of the aperture of a camera from which the objective has been removed, there is placed a diaphragm with a circular small prism with a small angle of deflection and placed excentrically with respect to the camera axis. The base of the prism is oriented towards the camera axis and is situated at such a distance from it as to let the parallel beam, hitting the prism parallel to the camera axis, fall, upon being deflected by the prism, at the centre of the photographic plate.

If $n$ such prisms be arranged around the camera axis, then the illumination at the centre of the photographic plate will be $n$ times greater than that given by a single prism.

For adjusting the prisms a small circular aperture with a slide is left in the centre of the diaphragm.

The camera axis is directed towards the Sun which gives a bright circle at the centre of the ground-glass. Hereupon the prisms are rotated in the plane of the diaphragm until the bright circles produced by them become concentric with that formed by the central aperture of the diaphragm.

I made use of two sets of four prisms in each, the angle of deflection being 42′. As should be expected, the exposure was reduced thereby about four times, so that I was able, in the spring of 1943, to obtain quite a few dozen of both subsidiary negatives and negatives for comparing the brightness and colour of Venus and Jupiter.

The observations were carried out with a four-camera coronograph, so that I could take photographs simultaneously in photographic and photovisual rays; this could be attained by using two cameras only.

In this way instead of an image, a certain even blackening is produced. A very important photometric feature of the method described consists in that by rotating the prisms the circles of illumination produced by them can be drawn apart, thus forming areas on which the illuminations are proportional to the figures 1, 2, 3...

The plate-holder is supplied with an insertion-piece movable in front of a rectangular mask, so that several photographs limited by the mask can be taken on a single plate. If for the brighter of the celestial bodies several photographs be taken with different exposures and for the less bright one, a single photograph with a longer duly selected exposure, then such a negative will bear both the scale of luminosity and the time-scale, *i. e.* all that is necessary for comparing photometrically two celestial bodies.

Even a single-image negative makes it possible to investigate the addition of photographic densities; if some of the prisms are supplied with screens, then it is possible to study the simultaneous photographic action of sources of different colour, which has been insufficiently investigated as yet.

Those properties of the method discussed make it extremely valuable for photometric purposes, and scarcely any other method can outmatch it. The only condition required by our method is that all the prisms be made of identical material and that they be kept absolutely clean.

Can the number of prisms be augmented? The prisms can be arranged in concentric rings with a centre at the camera axis; but the prisms of every next ring should have a greater angle of deflection, thus giving ever increasing spectra.

This implies certain inconveniences of both theoretical and technical nature. Besides, all the prisms within a given ring should with a rather high degree of accuracy be of an equal angle of deflection.

Much more convenient is the use of small aluminium-coated mirrors facing the sky by their glass-surface, which permits the latter to be safely cleansed *.

Every mirror is mounted on a circular plate. By means of one pulling and three pushing collimating screws every plate is attached to a common board.

Let us calculate a system of 50 small mirrors which collect light at a distance of 1000 mm from them. Let the diameter of each separate mirror be 27 mm. Each mirror would then form on the sky a solid angle of 1.88 square degrees. Fifty mirrors would increase the illumination on a photographic plate fifty times, which is equivalent to a luminosity from 94 square degrees; each separate mirror will reflect the light from the same sky area of 1.88 square degrees.

The total illumination given by all of the fifty mirrors will be 94 times greater than, or exceed by 4.9 magnitudes that of one square degree of the sky. One square degree of the sky is known, however, to correspond in brilliancy, on the average, to one star of the magnitude 4.8.

It is well known, moreover, that the integral light of the stars attains about $^1/_5$—$^1/_4$ of the total sky-light, made up of the star light and the luminosity of the upper layers of the atmosphere. In other words, the brightness of the night sky is by two star magnitudes greater still than that found from the stars alone, which corresponds to no less than one star of the magnitude 2.8 per one square degree.

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* Sets of mirrors up to 210 in number have long since been applied with the purpose of accumulating heat from the Sun by W. J. Buchman, head of the Astronomical Laboratory at the Kazakh State University.

Now what will be the efficiency of the 50 mirrors? As shown above, they will augment the brightness by 4.9 star magnitudes more; that is; one square degree will yield a light flux equal to that from a star (2.8—4.9) = —2.1 star magnitude, which approximates the brilliancy of Jupiter at the time of opposition.

As said above, my 1942 observations showed that direct illumination from Jupiter gives quite a sufficient darkening on the negative with an exposure of 15 minutes in photographic and 1.5 hours in photovisual rays. A system of 50 mirrors is thus quite sufficient for studying the brightness and colour of the whole sky, not to mention the average sky brightness in the Milky Way, where it is twice as great. Finally, the brightness spots in the Milky Way, for example, in Sagittarius, may be photographed with an exposure of about 5 minutes in photographic, and about 30 minutes in the photovisual rays.

It should be noted that the illumination produced on the plate is absolutely uniform and the measurement is thus reduced to merely determining the density of the blackening on the negative.

We thus obtain an extremely simple and inexpensive instrument, irreproachable in point of theory, for studying the brightness of the sky at any point, applicable to such problems as the average brightness and colour of the Milky Way at different points, selective light absorption in the dark nebulae of the Milky Way, *etc.* The only thing necessary is to secure two sets of fifty mirrors each, fixed on the mounting so as to permit the shifting of the one system relative to the other within 30 degrees in declination and by as much in right ascension.

This makes it possible to photograph two spots of the sky simultaneously, for example, a bright area in the Milky Way and a dark nebula.

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*PHYSICAL CHEMISTRY*

# ON THE NATURE OF INITIATING EXPLOSIVES

By K. K. ANDREEV

*(Communicated by N. N. Semenov, Member of the Academy, 2. XI. 1943)*

The use of explosives to-date has developed in accordance with the existence of two different types of substances which are described, respectively, as secondary and initiating explosives.

The causal mechanism underlying the different behaviour of these two types is not entirely clear.

The author's investigations ([1],[2]) have shown that secondary explosives burn at atmospheric pressure with a velocity that does not exceed 10 to 15 cm/min., whereas the velocity of burning of initiating explosives is, according to the data available, some hundred or thousand times greater ([3]).

The conclusion drawn from these observations was that so great a velocity in the case of initiating explosives must cause the pressure in the burning front to surpass the limits of a stationary process, thus leading to a more or less rapid initiation of detonation ([4]). In its turn the great velocity of burning was explained by the fact that for initiating explosives the value of the ratio $Q/E$, heat of reduction to activation energy, characterizing the ability of the reaction to self-propagation, is considerably greater than for secondary explosives ([5]).

This scheme, however, is in contradiction with some known facts. Nitroglycol, for instance, has a nearly equal activation energy (35 000 cal.) with lead azide, but its burning velocity is low and it does not possess initiating properties. On the other hand, lead styphnate, having a relatively great energy of activation ($\sim$46000 cal.), is nevertheless an initiator, although its initiating properties are not sharply expressed.

Moreover, if the low energy of activation, or, in other words, low stability of the molecule, were the only distinctive feature of initiating explosives, one might naturally expect that the flash-point, which to a certain extend characterizes the stability of the particular substance, would be lower for the initiators than for secondary explosives. But this is not true as a general rule.

In order to reconcile our interpretation with these facts factors responsible for the fundamental property of initiating explosives, their great burning velocity at atmospheric pressure, need to be more fully taken into account. Some of the author's experiments have shown that the burning velocity of explosives at low pressures may be expressed by the relation

$$V = A + f(P)$$

There is reason to suppose that the term $A$ in the first instance characterizes the relative rôle of the reaction in the condensed phase. The heat evolved by this reaction is transferred by conduction through the solid or liquid phase to the adjacent layers much more rapidly than in the case of volatile explosives, where

it is transferred to the surface of the burning explosive through a layer of poorly conducting vapours, because the reaction proceeds mainly in the gas phase.

Accordingly, in the case of nitroglycol which is a volatile explosive the term $A$ has the lowest value ($\sim 0$), and the velocity of burning at atmospheric pressure is correspondingly low. For nitrocotton which is not converted into gas by evaporation, the value of $A$ is large ($\sim 7.7$ g/cm$^2$ min.) and the velocity of burning is several times greater than in the case of nitroglycol.

Picric acid burns at $P=1$ kg/cm$^2$ with a velocity of about 1 cm/min.; the non-volatile potassium picrate possesses a 600 times greater velocity; and ammonium picrate which dissociates into gaseous products, burns with the velocity close to that of the picric acid.

Initiating explosives are characterized, as mentioned above, by a great velocity of burning even at atmospheric pressure. This is probably owing to a greater value of the term $A$, which means that the reaction in the solid phase plays a greater rôle here than in the case of secondary explosives.

The question naturally arises, What is the cause of this difference? Chemically the two types of compound do not belong to widely different classes, and many secondary explosives are known to yield initiating explosives.

This transformation is mainly carried out in two ways. The one is exemplified by the introduction of azide-groups in the molecule of trinitrobenzene to form trinitroazidobenzene, and the other by the introduction of metal in styphnic acid, whereby lead styphnate is formed.

In the former case the rise of the reaction velocity in the solid phase is due to a considerable decrease of the stability of the molecule, which among other things is characterized by a lowering of the flash-point. The volatility does not change considerably.

While trinitrobenzene when heated to a temperature corresponding to a considerable vapour pressure suffers no marked decomposition, and the chemical reaction proceeds mainly in the gaseous phase, trinitroazidobenzene decomposes rapidly even at temperatures at which its vapour pressure is very low.

In the latter case the stability of the molecule does not necessarily undergo a considerable change. But the vapour pressure diminishes strongly. Therefore there is no lowering of the flash-point, but at this temperature, corresponding to an intense decomposition of the explosive, the vapour density is low and the reaction proceeds as in the former case, mainly in the condensed phase.

It is easy to realize that with such a mechanism of propagation of burning and initiation of detonation, substances of simple structure, such as lead azide, which are not able of supplementary burning in the gaseous phase, possess more definitely initiating properties, as compared to compounds with a complex molecule.

In the light of these considerations a systematic study of the velocity of burning of initiating explosives as a function of pressure appears to be a matter of great interest.

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*PHYSICAL CHEMISTRY*

# COMPUTATION OF THE TEMPERATURE DEPENDENCE OF THE LOWER LIMIT OF IGNITION OF HYDROGEN FROM SPECTROSCOPIC MEASUREMENTS OF HYDROXYL

By V. KONDRATJEW, Corresponding Member of the Academy

Several years ago Avramenko [1] established by spectroscopic measurements that the concentration of hydroxyl in a rarefied hydrogen flame, determined by the absorption coefficient of individual OH lines, varies directly as the quantity $q$ of water formed in unit time

$$\mu l_i = Kq \qquad (1)$$

We will show that this result leads to a definite conclusion with regard to the temperature dependence of the lower limit of ignition of hydrogen, $p_1$, in agreement with direct measurements of this quantity.

The part played by hydroxyl in the oxidation of hydrogen reduces to its reaction with hydrogen proceeding according to the scheme:

$$OH + H_2 = H_2O + H \qquad (I)$$

with an energy of activation $E = 10\,000$ cal. per mole[2]. The hydrogen atoms forming therewith react with oxygen either according to the scheme

$$H + O_2 = OH + O \qquad (II)$$

involving branching of the reaction chains that are characteristic of the region of combustion, or according to the scheme

$$H + O_2 + M = HO_2 + M \qquad (III)$$

Process (II) which is considerably endothermic is postulated in connexion with the high temperature coefficient of the upper limit of ignition of hydrogen, $p_2$, to which there corresponds an energy of activation $E_2 \cong 24\,000$ cal. The trimolecular process (III) involves rupture of the chains in the gas phase. Here it is assumed that $HO_2$ under flame conditions is converted in the end into water either directly or through the hydrogen peroxide stage. Process (II) involves the branching process

$$O + H_2 = OH + H \qquad (IV)$$

as well, proceeding with an energy of activation $E_{IV} \cong 6\,000$ cal [3]. The large rate of process (IV), connected with the low value of the quantity $E_{IV}$, results in the O atoms perishing practically in the volume. The H atoms, on the contrary, in view of the low rates of processes (II) and (III), have time

to diffuse to the wall of the reaction vessel. In connexion with this, rupture of the chains on the walls due to absorption of H atoms is assumed according to the following scheme:

$$H \xrightarrow{\text{wall}} \frac{1}{2} H_2 \qquad (V)$$

Such, schematically, is the mechanism of the oxidation of hydrogen according to Elbe and Lewis([4]).

Employing this mechanism and applying Semenov's method of partial stationary concentrations ([5])

$$\left[\frac{d(OH)}{dt}=0;\ \frac{d(O)}{dt}=0;\ \frac{d(HO_2)}{dt}=0;\ \frac{d(H)}{dt}\neq 0\right]$$

we get

$$k_I(H_2)(OH)=2k_{II}(O_2)(H)=2k_{IV}(H_2)(O) \qquad (2)$$

$$\frac{d(H)}{dt}=\{[2k_{II}-k_{III}(M)](O_2)-k_V\}(H) \qquad (3)$$

$$\frac{d(H_2O)}{dt}=\left[2k_{II}+\frac{1}{2}k_{III}(M)\right](O_2)(H) \qquad (4)$$

where $k_I$, $k_{II}$ *etc.* are the rate constants of the corresponding processes.

If we divide (4) by (3) and replace $(O_2)$ by $(H_2O)$ on the basis of the equation

$$(O_2)_0=(O_2)+\frac{1}{2}(H_2O)$$

where $(O_2)_0$ is the initial concentration of oxygen in the mixture, an equation is obtained that can be integrated.

Performing this integration with the boundary condition $(H)=0$ when $(H_2O)=0$, we get

$$(H)=\frac{2k_{II}-k_{III}(M)}{2k_{II}+\frac{1}{2}k_{III}(M)}(H_2O)+\frac{2k_V}{2k_{II}+\frac{1}{2}k_{III}(M)}\ln\left[1-\frac{(H_2O)}{2(O_2)_0}\right] \qquad (5)$$

Confining ourselves to the case of the stoichiometric mixture $[2(O_2)_0=(H_2)_0]$ and introducing the quantity

$$\gamma=\frac{(H_2O)}{(H_2)_0}$$

which stands for that part of the hydrogen that has been converted into water, we get from (5) on the basis of (2)

$$(OH)=\frac{k_{II}}{k_I}\frac{1}{2k_{II}+\frac{1}{2}k_{III}(M)}\{[2k_{II}-k_{III}(M)](H_2)_0\gamma+2k_V\ln(1-\gamma)\} \qquad (6)$$

and then, from (4) and (5)

$$\frac{d\gamma}{dt}=\frac{1}{2}(1-\gamma)\{[2k_{II}-k_{III}(M)](H_2)_0\gamma+2k_V\ln(1-\gamma)\} \qquad (7)$$

Comparing (6) and (7), we find that

$$(OH)=-\frac{2k_{II}}{k_I}\frac{1}{2k_{II}+\frac{1}{2}k_{III}(M)}\frac{d\ln(1-\gamma)}{dt}$$

or

$$\int_0^t (OH)\,dt = -\frac{2k_{II}}{k_I}\,\frac{1}{2k_{II}+\frac{1}{2}k_{III}(M)}\ln(1-\gamma) \tag{8}$$

Let $\gamma_k$ stand for that part of the hydrogen that has reacted by the end of the process.

This quantity is obtained from the condition $\frac{d\gamma}{dt}=0$, whence on the basis of equation (7) we get

$$[2k_{II}-k_{III}(M)]\,(H_2)_0\,\gamma_k + 2k_V \ln(1-\gamma_k) = 0 \tag{9}$$

and, consequently,

$$\int_0^\tau (OH)\,dt = \frac{k_{II}}{k_I k_V}\,\frac{2k_{II}-k_{III}(M)}{2k_{II}+\frac{1}{2}k_{III}(M)}\,(H_2)_0\,\gamma_k = C\,(H_2O)_k \tag{10}$$

where

$$C = \frac{k_{II}}{k_I k_V}\,\frac{2k_{II}-k_{III}(M)}{2k_{II}+\frac{1}{2}k_{III}(M)}\;;\quad (H_2O)_k = (H_2)_0\,\gamma_k \tag{11}$$

and $\tau$ is the time that the mixture remains in the reaction vessel, *i. e.* the reaction time.

From equation (10) it is not difficult to get the empirical equation (1).

Considering the jet method employed by Avramenko [4], we find the quantity of water (in grams) forming in a unit of time to be

$$q = \frac{M}{N}\,S\,(H_2)_0 \int_0^l \frac{d\gamma}{dt}\,dl = \frac{M}{N}\,S\,\frac{\overline{dl}}{dt}\,(H_2O)_k \tag{12}$$

where $S$ is the cross-section of the reaction vessel; $l$ is the length of the reaction zone; $\frac{\overline{dl}}{dt}$ is the mean linear velocity of the gas; $M$ is the molecular weight of water and $N$ is the Avogadro number. On the other hand, the absorption coefficient $\mu l_i$ of a definite hydroxyl line is connected with its concentration by the equation

$$\mu l_i = \frac{i}{g''}\int_0^l \mu^0\chi\,(OH)\,dl \tag{13}$$

where $i$ is the intensity factor of the corresponding line; $g''$ is the statistical weight of the ground state of the absorbing molecule and $\chi$ is a quantity determining the fraction of OH molecules in the state $g''$. In view of the slight dependence of the product $\mu^0\chi$ upon the temperature (see below), we may rewrite equation (13) as follows:

$$\mu l_i = \frac{i}{g''}\,\mu^0\chi \int_0^l (OH)\,dl \tag{13'}$$

and then thus:

$$\mu l_i = \frac{i}{g''}\,\mu^0\chi \int_0^\tau \frac{dl}{dt}\,(OH)\,dt = \frac{i}{g''}\,\mu^0\chi\,\frac{\overline{dl}}{dt}\int_0^\tau (OH)\,dt \tag{14}$$

Expressing $\int_0^\tau (OH)\,dt$ by $\mu l_i$ on the basis of the last equation and $(H_2O)_k$ by

$q$ on the basis of equation (12), we get from (10):

$$\mu l_i = \frac{i}{g''} \mu^0 \chi C \frac{N}{SM} q = Kq \qquad (15)$$

where

$$K = \frac{i}{g''} \mu^0 \chi C \frac{N}{SM} \qquad (16)$$

The experimentally established proportionality between $\mu l_i$ and $q$ requires that the quantity $K$ should not depend either upon the temperature or upon the pressure in that range of variation of $T$ and $p$ which was considered in Avramenko's experiments ($T = 900 - 1\,400°$ K, $p = 10 - 40$ mm). This requirement imposes definite conditions upon the quantities entering into expression (16).

Let us have a look at what these conditions are like.

As to the pressure, it enters into the quantity $\mu^0$ as a broadening of the OH absorption lines due to collision, as well as into the quantity $C$, as follows directly from formula (11). However, at a pressure of 10—40 mm the width of the hydroxyl lines due to collision still constitutes a comparatively small part of their full width and it may be safely discarded, thus excluding the dependence of $\mu^0$ upon the pressure. We will show that the quantity $C$ likewise for all practical purposes does not depend upon the pressure.

And indeed, from the condition at the upper limit of ignition that

$$2k_{II} = k_{III}(M)_2 \qquad (17)$$

we get

$$\frac{k_{III}(M)}{2k_{II}} = \frac{(M)}{(M)_2} = \frac{p}{p_2} \qquad (18)$$

By calculating from Nalbandjan's (6) data the quantity $\frac{p}{p_2}$ for different pressures and temperatures in the $T$ and $p$ interval pointed out above, it can be shown that in this interval the factor

$$\frac{k2_{II} - k_{III}(M)}{2k_{II} + \frac{1}{2} k_{III}(M)} = \frac{1 - p/p_2}{1 + \frac{1}{2} p/p_2}$$

is practically equal to 1, *i. e.* the quantity $C$ which we may now rewrite in the form

$$C = \frac{k_{II}}{k_I k_V} \qquad (11')$$

is likewise found to be independent of the pressure.

We pass on to the computation of the temperature dependence of the quantity $K$.

The factor $\frac{i}{g''} \mu^0 \chi$ entering into this quantity on the basis of spectroscopic data (7) in the temperature interval 900—1 400° K may be represented with an accuracy of several per cent in the form

$$\frac{i}{g''} \mu^0 \chi = \frac{A_0}{T} \qquad (19)$$

where $A_0$ is a numerical coefficient. Then we transform the quantity $C$ as follows.

Determining the quantity $\frac{2k_{II}}{k_V}$ from the condition at the lower limit of ignition

$$2k_{II}(O_2)_1 = k_V \tag{20}$$

in the following way:

$$\frac{2k_{II}}{k_V} = \frac{1}{(O_2)_1} = \frac{3}{(M)_1} = \frac{3kT}{p_1} \tag{21}$$

and representing the constant $k_I$ by the formula

$$k_I = \varepsilon_I 2\sqrt{2\pi}\, d^2 \sqrt{\frac{RT}{\mu}}\, e^{-\frac{E_I}{RT}} \tag{22}$$

where $\varepsilon_I$ is the steric factor, $d$ is the half-sum of the diameters of the OH and $H_2$ molecules and $\mu$ is their reduced mass, we get:

$$C = \frac{3\sqrt{R}}{4\sqrt{2\pi N}} \frac{\sqrt{\mu}}{d^2} \frac{1}{\varepsilon_I} \frac{\sqrt{T} e^{E_I/RT}}{p_1} \tag{23}$$

Substituting (23) and (19) into expression (16), we shall have

$$K = \frac{3\sqrt{R}}{4\sqrt{2\pi}} A_0 \frac{\sqrt{\mu}}{d^2 SM} \frac{1}{\varepsilon_I} \frac{e^{E_I/RT}}{\sqrt{T} p_1} \tag{24}$$

As we see, the requirement that the quantity $\frac{e^{E_I/RT}}{\sqrt{T} p_1}$ be independent of the temperature follows from the experimentally established constancy of the quantity $K$; $p_1$ is usually approximated in the form

$$p_1 = a_1 e^{E_1/RT} \tag{25}$$

That is why the foregoing conclusion reduces to the condition

$$\frac{e^{(E_I - E_1)/RT}}{\sqrt{T}} = \text{const} \tag{26}$$

in the temperature interval 900—1400° K. Taking logarithms and then differentiating expression (26) we get $E_1 - E_I = \frac{R}{2} T$, whence it follows that the closer the difference $E_1 - E_I$ is to $T$, the better condition (26) is satisfied. And since $T$ is a quantity of the order of 1000, we find that $E_1 - E \cong 1000$ cal. and, consequenlty, that

$$E_1 \cong 10000 + 1000 = 11000 \text{ cal.}$$

This result is in complete agreement with the experimental data of Nalbandjan (6), according to which in vessels made of diverse materials with walls treated in different ways the pressure at the lower limit of ignition of hydrogen is expressed by formula (25) with $E_1 = 11000$ cal.

In conclusion let us consider the following, very essential circumstance. As follows from the theory of the lower limit of ignition, under the condition of Nalbandjan's and Avramenko's experiments, $p_1$ should be proportional to the quantity $\frac{\varepsilon_V}{k_{II}}$, *i. e.* to $\varepsilon_V e^{24000/RT}$, where $\varepsilon_V$ is the probability of the

adsorption of active centres (of hydrogen atoms) by the wall. This relation and the formula $p_1 = a_1 e^{11000/RT}$ are compatible, evidently, only if it is assumed that

$$\varepsilon_V \sim e^{11000/RT} : e^{24000/RT} = e^{-13000/RT}$$

*i. e.* upon the assumption of a marked temperature dependence of the quantity $\varepsilon_V$ expressed in its rapid rise with an increase in temperature. However, whether we are dealing here with activated adsorption, or whether the temperature dependence of $\varepsilon_V$ pointed out above is due to a similar temperature dependence of the free active surface of the adsorbent, cannot be decided without setting up special experiments.

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*GEOLOGY*

# NOTES ON ANALYTICAL GEOLOGY

By A. B. VISTELIUS

*(Communicated by W. I. Vernadsky, Member of the Academy, 1. II. 1944)*

1. The aim of the work was to bring out the behaviour of the porosity coefficient in a block of Spirifer deposits exposed by a number of bore-holes in the western part of the Buguruslan petroliferous region.

This paper, in the first place, is intended to show that the correlation between the space held in the stratisphere by the solid phase, and the two other phase states of matter, as expressed by the porosity coefficient, is a property typical of the deposits studied; secondly, its purpose is to give an illustration, were it even on a modest scale, of the applicability of the analytical method to the study of geological phenomena.

2. When collecting materials on the physical properties of the oil-bearing deposits of the Permian, the author received from N. E. Frolova, petrographer, some data on the determination of the porosity coefficient in specimens from seven drill-holes exposing the Spirifer deposits in the region of the Amanak, Kalinov-Zhuravlev and Novo-Stepanov structures of the Buguruslan oil-bearing region. The depths of sampling were fixed precisely; all the determinations of the porosity coefficient were made at the same laboratory by the same person, with the aid of the same apparatus.

3. The method for analysing the data consisted in the following: in a rectilinear, rectangular system of coordinates $(x, y)$ along the $x$ axis were plotted the depths of sampling; and along the $y$ axis, the values of the porosity coefficient. After all data concerning a given hole had been plotted, the adjacent points were interpolated linearly, and the broken line thus obtained was smoothed out after the parabole

$$u = \frac{1}{35}\{17u_0 + 12(u_1 + u_{-1}) - 3(u_2 + u_{-2})\} \qquad (1)$$

where $u$ is the smoothed value of the ordinate; $u_0$, the ordinate of the middle of the area smoothed out; $u_{1\ (-1)}$ and $u_{2\ (-2)}$, the extreme ordinates, the internal and the external, respectively, of the same area.

For the procedure of smoothing we made use of N. N. Mikhailov's [1] nomographs. According to the points obtained after smoothing an empirical formula* was deduced by the graphic method for each individual hole; the parameters of this were plotted on the map at the points of location of the corresponding holes.

* The empirical formulae, obtained for the sections exposed by drill-holes Nos. 3 and 6, were treated by the least square method; but further on the treatment by this method was abandoned, and the corrections obtained were not introduced into the formula, since the very nature of the phenomenon studied and the intervals between the specimens yielded an accuracy no less than the one attained with the aid of the method here described [2].

## Table 1

### Hole 1

| h | x | y |
|---|---|---|
| 309.0 | 20.90 | 20.68 |
| 311.0 | 22.90 | 2.44 |
| 313.0 | 24.90 | 3.63 |
| 315.0 | 26.90 | 2.49 |
| 317.0 | 28.90 | 4.31 |
| 319.0 | 30.90 | 15.14 |
| 321.0 | 32.90 | 15.92 |
| 325.0 | 36.90 | 4.41 |
| 329.0 | 40.90 | 21.46 |
| 333.0 | 44.90 | 5.33 |
| 335.0 | 46.90 | 13.90 |
| 337.0 | 48.90 | 12.05 |

### Hole 2

| h | x | y |
|---|---|---|
| 264.0 | 2.00 | 17.48 |
| 267.0 | 5.00 | 29.99 |
| 268.0 | 6.00 | 23.91 |
| 269.3 | 6.30 | 8.55 |
| 270.3 | 8.30 | 5.02 |
| 271.8 | 9.80 | 25.11 |
| 273.5 | 11.50 | 26.82 |
| 275.4 | 13.40 | 16.88 |
| 276.4 | 14.40 | 23.29 |
| 278.0 | 16.00 | 20.00 |
| 279.8 | 17.80 | 29.45 |
| 286.0 | 24.00 | 17.24 |
| 288.0 | 26.00 | 6.68 |
| 289.0 | 27.00 | 4.77 |
| 290.0 | 29.00 | 1.00 |
| 291.0 | 29.00 | 7.17 |
| 292.0 | 30.00 | 35.80 |
| 293.2 | 31.20 | 27.92 |
| 294.2 | 32.20 | 31.22 |
| 295.0 | 33.00 | 25.54 |
| 296.2 | 34.20 | 32.28 |
| 297.2 | 35.20 | 25.61 |
| 298.2 | 36.20 | 27.00 |
| 299.0 | 37.00 | 13.00 |
| 300.0 | 38.00 | 25.00 |
| 303.0 | 41.00 | 18.20 |
| 304.0 | 42.00 | 5.96 |
| 305.0 | 43.00 | 14.30 |
| 306.0 | 44.00 | 5.00 |
| 307.0 | 45.00 | 15.00 |
| 308.0 | 46.00 | 12.00 |
| 310.0 | 48.00 | 1.00 |
| 311.0 | 49.00 | 11.70 |

### Hole 3

| h | x | y |
|---|---|---|
| 277.4 | 7.50 | 26.50 |
| 279.2 | 9.30 | 25.24 |
| 281.4 | 10.50 | 23.59 |
| 282.6 | 11.70 | 16.40 |
| 283.0 | 12.10 | 26.39 |
| 284.3 | 13.40 | 34.47 |

(continued)

| h | x | y |
|---|---|---|
| 284.6 | 13.70 | 1.84 |
| 286.2 | 15.30 | 4.61 |
| 288.0 | 17.10 | 3.27 |
| 288.0 | 17.10 | 6.00 |
| 289.3 | 18.10 | 7.23 |
| 291.0 | 20.10 | 18.68 |
| 293.0 | 22.10 | 5.10 |
| 294.0 | 23.10 | 24.83 |
| 296.0 | 25.10 | 27.08 |
| 297.0 | 26.10 | 9.84 |
| 297.0 | 26.10 | 15.86 |
| 298.0 | 27.10 | 3.92 |
| 300.0 | 29.10 | 19.25 |
| 305.0 | 34.10 | 0.49 |
| 308.0 | 37.10 | 21.99 |
| 309.0 | 38.10 | 14.38 |
| 311.0 | 40.10 | 13.30 |
| 312.0 | 41.10 | 26.85 |
| 313.0 | 42.10 | 3.74 |
| 316.5 | 45.60 | 1.63 |
| 320.0 | 49.10 | 2.08 |
| 324.0 | 53.10 | 13.12 |

### Hole 4

| h | x | y |
|---|---|---|
| 258.0 | 2.00 | 22.00 |
| 263.0 | 7.00 | 22.70 |
| 265.0 | 9.00 | 19.10 |
| 268.0 | 12.00 | 12.00 |
| 270.0 | 14.00 | 17.30 |
| 273.0 | 17.00 | 14.30 |
| 275.0 | 19.00 | 13.70 |
| 278.0 | 22.00 | 26.90 |
| 280.0 | 24.00 | 18.20 |
| 283.0 | 27.00 | 10.10 |
| 288.0 | 32.00 | 31.40 |
| 293.0 | 37.00 | 13.70 |
| 298.0 | 42.00 | 9.30 |
| 303.0 | 47.00 | 21.30 |
| 308.0 | 52.00 | 26.40 |
| 319.0 | 57.00 | 20.20 |
| 319.0 | 63.00 | 8.60 |
| 323.0 | 67.00 | 13.80 |

### Hole 5

| h | x | y |
|---|---|---|
| 291.0 | 16.85 | 13.00 |
| 293.0 | 18.85 | 3.20 |
| 295.0 | 20.85 | 17.90 |
| 297.0 | 22.85 | 1.00 |
| 302.0 | 27.85 | 13.00 |
| 307.0 | 32.85 | 6.00 |
| 310.0 | 35.85 | 11.80 |
| 316.0 | 41.85 | 2.90 |
| 320.0 | 45.85 | 2.60 |
| 323.0 | 48.85 | 3.80 |
| 329.0 | 54.85 | 15.30 |
| 331.0 | 56.85 | 17.20 |
| 333.0 | 58.85 | 0.90 |

(continued)

### Hole 6

| h | x | y |
|---|---|---|
| 278.0 | 3.00 | 18.30 |
| 280.0 | 5.00 | 20.80 |
| 281.0 | 6.00 | 21.70 |
| 282.0 | 7.00 | 25.70 |
| 283.0 | 8.00 | 30.80 |
| 284.0 | 9.00 | 33.60 |
| 285.0 | 10.00 | 35.20 |
| 286.0 | 11.00 | 15.50 |
| 288.0 | 13.00 | 16.10 |
| 290.0 | 15.00 | 5.60 |
| 292.0 | 17.00 | 15.80 |
| 294.9 | 19.90 | 17.50 |
| 296.0 | 21.00 | 21.50 |
| 298.0 | 23.00 | 7.20 |
| 300.0 | 25.00 | 18.60 |
| 302.0 | 27.00 | 28.20 |
| 304.0 | 29.00 | 15.10 |
| 306.0 | 31.00 | 22.50 |
| 308.0 | 33.00 | 23.90 |
| 310.0 | 35.00 | 13.80 |
| 312.0 | 37.00 | 23.00 |
| 314.0 | 39.00 | 22.60 |
| 316.0 | 41.00 | 22.90 |
| 318.0 | 43.00 | 20.40 |
| 320.0 | 45.00 | 14.60 |
| 322.0 | 47.00 | 16.90 |
| 324.0 | 49.00 | 9.90 |
| 326.0 | 51.00 | 20.20 |
| 328.0 | 53.00 | 16.10 |
| 330.0 | 55.00 | 13.00 |

### Hole 7

| h | x | y |
|---|---|---|
| 249.0 | 0.20 | 12.90 |
| 250.0 | 1.20 | 16.13 |
| 251.0 | 2.20 | 16.25 |
| 252.0 | 3.20 | 16.70 |
| 253.0 | 4.20 | 17.40 |
| 254.0 | 5.20 | 17.40 |
| 255.0 | 6.20 | 16.90 |
| 256.0 | 7.20 | 17.40 |
| 258.0 | 9.20 | 21.85 |
| 260.0 | 11.20 | 26.60 |
| 261.0 | 12.20 | 8.00 |
| 262.0 | 13.20 | 9.70 |
| 263.0 | 14.20 | 6.50 |
| 264.0 | 15.20 | 7.20 |
| 265.0 | 16.20 | 2.50 |
| 266.0 | 17.20 | 1.70 |
| 267.0 | 18.20 | 10.90 |
| 268.0 | 19.20 | 10.00 |
| 269.0 | 20.20 | 0.65 |
| 270.0 | 21.20 | 6.90 |
| 271.0 | 22.20 | 9.60 |
| 272.0 | 23.20 | 8.00 |
| 273.0 | 24.20 | 24.80 |
| 274.0 | 25.20 | 3.00 |
| 275.0 | 26.20 | 5.10 |
| 275.95 | 27.15 | 7.10 |

Table Ia

Hole 1

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 20.90 | +2.5 | +1.0 | +1.5 |
| 23.90 | +2.0 | −2.0 | 0.0 |
| 26.90 | −7.0 | −7.0 | 0.0 |
| 29.90 | −2.0 | −1.5 | −0.5 |
| 32.90 | +1.0 | 0.0 | +1.0 |
| 35.90 | −3.0 | −3.5 | +0.5 |
| 38.90 | −1.0 | −2.0 | +1.0 |
| 41.90 | +4.0 | +4.5 | −0.5 |
| 44.90 | +1.5 | +2.0 | −0.5 |
| 47.90 | +0.5 | −1.0 | +1.5 |
| 50.90 | +0.5 | +1.0 | +0.5 |

Hole 2

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 2.00 | +2.5 | +7.5 | −5.5 |
| 5.00 | +3.0 | +4.0 | −1.0 |
| 8.00 | −2.0 | +2.0 | −4.0 |
| 11.00 | +2.0 | +0.5 | +1.5 |
| 14.00 | +6.0 | +6.0 | 0.0 |
| 17.00 | +7.0 | +5.5 | +1.5 |
| 20.00 | +4.0 | +1.5 | +2.5 |
| 23.00 | −1.0 | −0.5 | −0.5 |
| 26.00 | −6.0 | −4.0 | −2.0 |
| 29.00 | 0.0 | −1.0 | +1.0 |
| 32.00 | +7.0 | +4.0 | +3.0 |
| 35.00 | +6.0 | +6.0 | 0.0 |
| 41.00 | −1.0 | +5.0 | −6.0 |
| 44.00 | −8.0 | −9.5 | +1.5 |
| 47.00 | −3.0 | −1.5 | −1.5 |

Hole 3

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 7.50 | + 6.5 | +6.5 | 0.0 |
| 10.50 | + 6.5 | +8.0 | −1.5 |
| 13.50 | + 4.5 | +1.5 | +2.5 |
| 16.50 | − 6.0 | −4.0 | −2.0 |
| 19.50 | − 5.0 | +0.5 | −1.0 |
| 22.50 | − 0.5 | +2.5 | −3.0 |
| 25.50 | + 3.5 | +0.5 | +0.5 |
| 28.50 | − 1.5 | +3.5 | −5.0 |
| 31.50 | − 3.0 | −2.5 | −0.5 |
| 34.50 | − 4.5 | −7.0 | +2.5 |
| 37.50 | − 5.0 | −4.0 | +1.0 |
| 40.50 | + 2.0 | +2.0 | 0.0 |
| 46.50 | − 7.0 | −4.0 | −3.0 |
| 49.50 | −10.5 | −9.0 | −1.5 |
| 52.50 | − 6.0 | −9.0 | +3.0 |

Hole 4

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 11.0 | −3.0 | +0.5 | −3.5 |
| 15.5 | +1.0 | −2.0 | +3.0 |
| 20.0 | +1.0 | −2.0 | +3.0 |
| 24.5 | +4.0 | +2.0 | +2.0 |

(continued)

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 29.0 | +0.5 | − 0.5 | +1.9 |
| 33.5 | 0.0 | − 5.0 | +5.0 |
| 38.0 | −5.0 | −10.0 | +5.0 |
| 42.5 | −0.5 | 0.0 | −0.5 |
| 47.0 | +6.5 | + 5.5 | +1.0 |
| 51.5 | +7.0 | + 6.0 | +1.0 |
| 56.0 | 0.0 | + 2.0 | −2.0 |
| 60.5 | −2.0 | − 2.0 | 0.0 |
| 69.5 | −5.5 | − 2.5 | −3.0 |
| 74.0 | −1.0 | − 0.5 | −0.5 |

Hole 5

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 16.85 | +2.5 | −2.5 | +5.0 |
| 19.85 | +1.0 | +0.5 | +0.5 |
| 22.85 | −2.5 | −1.5 | −1.0 |
| 25.85 | −0.5 | −1.0 | +0.5 |
| 28.85 | +1.5 | +2.0 | −0.5 |
| 31.85 | +1.0 | +4.5 | −3.5 |
| 34.85 | +0.5 | +2.0 | −1.5 |
| 37.85 | 0.0 | +4.0 | −4.0 |
| 39.85 | −2.0 | 0.0 | −2.0 |
| 42.85 | −4.0 | −3.5 | −0.5 |
| 45.85 | −3.5 | −6.0 | +2.5 |
| 48.85 | −0.5 | +2.5 | −3.0 |
| 54.85 | +5.0 | +3.5 | +1.5 |
| 57.85 | +2.5 | +0.5 | +2.0 |

Hole 6

| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 3.00 | +5.5 | +7.5 | −2.5 |
| 6.00 | +8.5 | +5.5 | +3.0 |
| 9.00 | +2.0 | +1.0 | +1.0 |
| 12.00 | −4.0 | +2.0 | −2.0 |
| 15.00 | −3.0 | −3.0 | 0 |
| 18.00 | −2.5 | −3.0 | +0.5 |
| 21.00 | 0 | +0.5 | −0.5 |
| 24.00 | +2.5 | +2.0 | +0.5 |
| 27.00 | +2.5 | +2.5 | −0.5 |
| 30.00 | +2.0 | +3.0 | −1.0 |
| 33.00 | +2.5 | +3.5 | −1.0 |
| 36.00 | +2.5 | +3.0 | −0.5 |
| 42.00 | −4.0 | −1.5 | −2.5 |
| 45.00 | −4.0 | −4.5 | +0.5 |
| 48.00 | −1.5 | −6.5 | +5.0 |
| 51.00 | −2.0 | −1.0 | −1.0 |

Hole 7

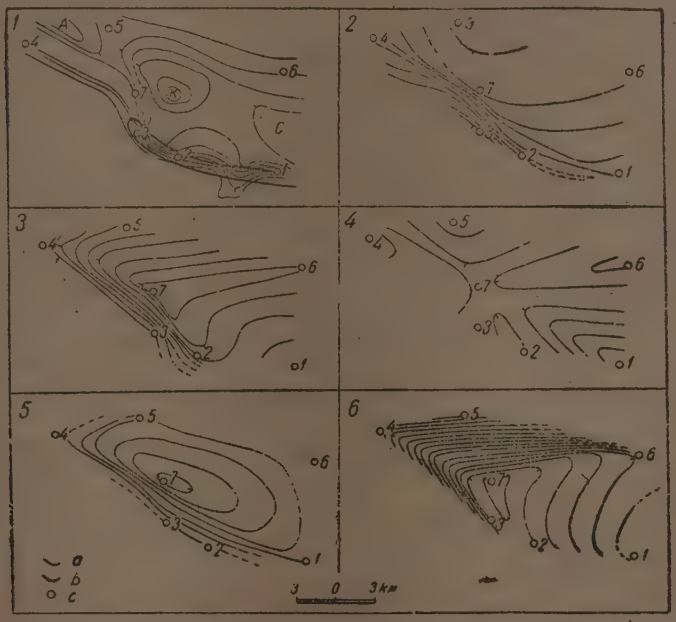
| $x$ | $y_0$ | $y_1$ | $\Delta y$ |
|---|---|---|---|
| 0.20 | +0.5 | +0.5 | 0.0 |
| 3.20 | +3.0 | +2.5 | +0.5 |
| 6.20 | +4.0 | +4.0 | 0.0 |
| 9.20 | +2.0 | +2.0 | 0.0 |
| 12.20 | −2.5 | −2.5 | 0.0 |
| 15.20 | −5.0 | −6.0 | +1.0 |
| 18.20 | −7.0 | −5.5 | −1.5 |
| 21.20 | −2.5 | −1.0 | −1.5 |
| 24 20 | +3.5 | +3.5 | 0.0 |
| 27.20 | 0.0 | +2.0 | −2.0 |

4. All the values of the porosity coefficient obtained from observations made for each hole fit within a formula of the general type:

$$y = m + nx + e^{a_1 x + b_1} \cos(\omega_1 x - \varphi_1) + e^{a_2 x + b_2} \cos(\omega_2 x - \varphi_2) \quad (2)$$

where $y$ is the value of the porosity coefficient in per cent; $x$, depth in meters, measured from the roof of the Spirifer deposits; $m$, mean value of the porosity coefficient, starting from which the calculation is made; $n$, coefficient at the linear member; $a_1$ and $a_2$, coefficients of extinction; $b_1$ and $b_2$, parameters characterizing the intensity; $\omega_1$ and $\omega_2$, periods; $\varphi_1$ and $\varphi_2$, initial phases of the process determining the value of the property investigated.

Table 1 presents the values of the porosity coefficient ($y$) obtained at the




*a*, isolines drawn at equal intervals; *b*, outlines of the zone enriched with liquid hydrocarbons; *c*, holes with a studied porosity
*1*. Sketch map of the region of the Amanak (*A*), Kalinov-Zhuravlev (*K*) and N.-Stepanov (*C*) structures according to the data available. 2. Sketch map compiled according to data for holes with a studied porosity. 3. Isolines of extinction coefficient $a_2$. 4. Isolines of coefficient $\omega_1$, 5. Isolines of coefficient $b_2$. 6. Isolines of ratio of coefficients $\omega_1$ to $\omega_2$

laboratory; column $h$ gives the depths of sampling, measured from the mouth of the holes, and column $x$, the same depths taken starting from the roof of the Spirifer deposits.

Table 1a shows the values of the porosity coefficient $y_0$, obtained upon the smoothing of the curve, the values of the same coefficient, as calculated by formula (2), $y_1$ and $\Delta y = y_0 - y_1$.

The values of the parameters of equation (2) are given in Table 2.

After having found out the functional relations between the depth of occurrence of the rock and the porosity coefficients; the values of all the parameters obtained for individual holes were mapped in the way mentioned above.

The schemes show the location of the drill-holes studied relatively to one another.

Table 2

| Hole No. | $m$ | $n$ | $a_1$ | $b_1$ | $\omega_1$ | $\varphi_1$ | $a_2$ | $b_2$ | $\omega_2$ | $\varphi_2$ | $\omega_1/\omega_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.3 | +0.250 | —0.0483 | +2.176 | 35.30 | 38.97 | +0.0136 | +0.913 | 28.47 | 235.02 | 1.24 |
| 2 | 21.7 | —0.095 | —0.0220 | +1.934 | 19.36 | 66.76 | +0.0220 | +1.403 | 19.36 | 105.42 | 1.00 |
| 3 | 17.8 | —0.190 | —0.0152 | +1.241 | 21.42 | 186.36 | +0.0066 | +1.398 | 21.42 | 199.25 | 1.00 |
| 4 | 16.0 | +0.034 | —0.0260 | +1.779 | 16.29 | 110.08 | +0.0044 | +1.258 | 10.11 | 59.02 | 1.62 |
| 5 | 20.6 | —0.035 | —0.0150 | +1.968 | 16.35 | 197.35 | +0.0260 | +0.981 | 14.12 | 110.16 | 1.16 |
| 6 | 8.2 | 0.000 | —0.1014 | +3.104 | 27.34 | 182.68 | +0.0120 | +0.816 | 16.00 | 351.20 | 1.71 |
| 7 | 14.5 | —0.241 | +0.0227 | +1.428 | 20.00 | 124.00 | +0.0287 | +0.090 | 21.82 | 95.96 | 0.92 |

5. Summing up the results of the above-described investigation, the following statement can be made:

a) Porosity is a property obeying definite laws, which is subject to periodical changes in the vertical sections of the deposits studied.

b) When plotted on the map, the parameters of the porosity equation are found to be related with the structure of the given region (schemes 3 and 4) and with the outlines of the zone enriched in liquid hydrocarbons (schemes 1 and 6).

c) From a comparison of the schemes shown in the figure it is evident that the outlines of some structures with an equally small number of holes can be mapped more precisely after the parameters of the porosity equation (scheme 5) than directly after the markers (scheme 2).

d) The areas within the outlines formed by the isolines of the parameters of equation (2), repeating the outlines of the better pronounced structure (schemes 3 and 5) are larger than the areas within the structures themselves (scheme 1).

In this way, the distribution of the porosity coefficients in the deposits investigated follows a periodic law, according to which the parameters of the porosity coefficient do not depend on the depth, but are in a certain manner related to the location of the point investigated relative to the structure and, possibly, to the area enriched with liquid hydrocarbons.

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*PALAEONTOLOGY*

# ON THE DENTITION OF *FLEURANTIA DENTICULATA*

By A. P. BYSTROW

*(Communicated by A. A. Borissiak Member of the Academy, 24. XII. 1943)*

In 1937, Graham-Smith and Westoll(1) published a paper on a new dipnoan fish, *Fleurantia denticulata*. In that paper the authors gave a description and interpretation of some peculiarities of the dentition of that dipnoan.

Four rows of rather large teeth were detected on the pterygoid of *Fleurantia*. These teeth are arranged fan-like in radial rows, quite like in *Dipterus valenciennesii*. Besides these large teeth the whole palatinal surface of the pterygoid is covered with irregularly scattered small teeth which habitually are called shagreen denticles (see figure).

The presence of shagreen denticles and the separation of the large pterygoid teeth from one another should, of course, be considered as primitive features in *Fleurantia*. It is, however, not this circumstance that calls for our comment.

Graham-Smith and Westoll pointed out the following detail in the general pattern of the pterygoid teeth in that fish. In the mesial parts of each row of large teeth they detected circles which they rightly interpreted as traces of shed teeth. Without doubt, of great interest is the presence of denticles at the bottom of these circles, as noted by the authors. These denticles are not larger than the usual shagreen denticles of the same pterygoid. In the first row there is only one circle in which one denticle is set. In the second row, four circles; in one of them three denticles are inserted; in the rest, one denticle in each. In the third row three circles are present: the first one without denticles; the second, with two; and the third, with one. In the fourth row, one denticle in each of the two circles is observed.

The authors regard this picture to be a case of replacement of teeth. They admit that young denticles might have caused the shedding of old teeth by their pressure.

Such an explanation of the replacement of teeth can be made only on the assumption of the young denticles arising under the bases of still functioning old teeth.

The evidence available to us on the structure and replacement of shagreen denticles in a number of fossil forms (2, 3) make us conclude that such an interpretation of the replacement of teeth in *Fleurantia* is quite incorrect.

There is no doubt that not one of the denticles observed in the circles on the surface of the pterygoid of *Fleurantia* did or could arise under an old large tooth.

Therefore the denticles could not cause the shedding of old teeth by their growth-pressure. Such a replacement of teeth was never observed in *Crossopterygii* or in *Dipnoi*. The denticles detected in the circles of the bone surface could arise there only after, not before the shedding of the large teeth. Likewise, it is certain that the denticles can on no account be interpreted as young pterygoid teeth replacing the old ones.

In my opinion, the picture described by Graham-Smith and Westoll is to be interpreted only in the following way.

In the course of the growth of the skull of the fish the dimensions of its pterygoid gradually increased. Owing to this, the lateral margin of that bone advanced outwards. At the same time, the pterygoid thickened by means of apposition of new layers of bone on its ventral as well as dorsal surface. This mode of growth is responsible for certain peculiarities in the replacement of teeth.

After the lateral margin of the pterygoid had sufficiently increased, a new pterygoid tooth was added to each row of large teeth on its ventral surface. These new teeth were somewhat larger than their predecessors. This difference in dimensions was proportional to the slight enlargement of the fish skull during the corresponding period of time. Thus, gradually the dental rows, as it were, followed the advancing margin of the growing pterygoid, and the teeth of each next generation were larger than the teeth of the preceding generation. Therefore we see in the rows of the preserved pterygoid teeth of *Fleurantia* that the teeth become the larger, the more laterally they are situated.

Along with the appearance of new teeth the oldest ones, situated at the mesial end of each row, were shed, leaving after them circles on the bone surface due to bone resorption. There is no doubt that in *Fleurantia* the shed large teeth were not always replaced by true pterygoid teeth. Often the shagreen denticles settled in their place as well as in other spots of the palatinal surface of the pterygoid.

The replacement of these shagreen denticles occurred continuously and in the course of the thickening of the bone they gradually became enclosed in it. As a result, the generations of shagreen denticles substituting one another became, as it were, immersed in the bony substance of the pterygoid. Had Graham-Smith and Westoll prepared a section of the pterygoid of *Fleurantia*, they would doubtless have discovered in its substance the earlier generations of shagreen denticles, *i. e.* obtained the picture described in a number of *Crossopterygii* ([3]) and *Labyrinthodontia* ([2]).

Thus, the shed large pterygoid teeth were replaced in each row by small shagreen denticles. These coalesced with the surface of the pterygoid, irrespective of whether this particular spot was or was not previously occupied by a large tooth. Strange as it may seem at first glance, this accounts for the fact that in some cases a circle left by a shed pterygoid tooth was occupied not by one, but by two or even three shagreen denticles, or was left free at all.

Only such an interpretation of the facts described by Graham-Smith and Westoll is in accordance with the details known on the replacement of the pterygoid and shagreen teeth.

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*PLANT PHYSIOLOGY*

# ON THE RELATION BETWEEN LEAF DEVELOPMENT AND FORMATION OF LATICIFEROUS VESSELS IN ROOTS OF KOK-SAGHYZ AND KRYM-SAGHYZ

By A. A. NICHIPOROVICH and E. F. IVANITZKAJA

*(Communicated by A. N. Bach, Member of the Academy, 8. VIII. 1943)*

The only rubber-containers in the root of kok-saghyz and krym-saghyz are the laticiferous vessels. The degree of their development and the extent to which they pervade the roots is therefore the main factor responsible for the rubber content in the row material. The laticiferous vessels of the rubber-plants listed above are segmented structures, which points to their secondary origin ([1]). Their formation in the secondary cortical tissue of the root is effected by way of differentiation and adherence of those cells which are deposited by cambium in the centrifugal direction in the process of phloem formation. The cambium encircles the central pith cylinder, the root of kok-saghyz thus being characterized by a regular monocentrical structure. The laticiferous vessels are arranged in the bark as concentric circles. They are interconnected within each circle by numerous anastomoses, while the laticiferous vessels of different circles are not connected with one another through anastomoses ([2–5]). The degree to which a root is pervaded by laticiferous vessels, and, consequently, its rubber-content, is determined by the following factors: 1) abundance of laticiferous vessels within every circle; 2) cross-section of the vessels; 3) degree of filling of the vessels with rubber, or concentration of the latter in the latex; 4) rate of differentiation of cells of bark parenchyme formed by cambium into laticiferous vessels, this process being attended by a periodical formation of concentrical circles of laticiferous vessels.

The highly variable rubber content in kok-saghyz depends largely upon the genotypical and phenotypical variation of the first three factors. The fourth character, *viz.* the rhythm of the formation of the concentrical circles of laticiferous vessels, is, on the contrary, extremely stable: between two adjoining circles of laticiferous vessels one usually can count up to 6—7, 8—9 or 10—11 intermediate parenchymatous cells, according to the age of the root. This relation is observed in roots showing most various degrees of rubber content.

When analysing this phenomenon, two fundamental questions arise:

1. What is responsible for the persistent stability of the rhythm of the formation of the circles of laticiferous vessels?

2. Is this rhythm absolutely stable indeed, or perhaps it may be shifted and altered in a way desirable to us?

When approaching the former question, we cannot help to assume that there is a definite relation between the formation of leaves and that of the circles of laticiferous vessels. And this supposition seems the more probable at first sight, as it is just in the course of leaf formation that cyclicity is observable. The greatly shortened stalks of dandelions show a spiral leaf-arrangement, and in

the process of the plant's growth the completion of each cycle of leaf-arrangement might correspond to the formation of a new circle of laticiferous vessels.

If this be so, taking into account the stability of the mode of leaf-arrangement in all plants belonging to a definite species, we should in all cases observe a constant relation between the number of leaves and that of the circles of laticiferous vessels; for briefness sake this relation will be denoted in the subsequent through L : V. The only rather scanty material bearing on the problem is available in a paper by Savchenko (6), but it is insufficient for its solution. We investigated the relation between the number of leaves and that of circles of laticiferous vessels, and the results of this work are summarized in this note.

The kok-saghyz plants to be analysed were taken from the plots of the Kirghizian Breeding station on July 17, September 9 and October 23, those of krym-saghyz on July 26, September 9 and October 17. The kok-saghyz seedlings failed to sprout at one, and the first sampling included therefore such plantlets as differed essentially as to their age.

Table 1 gives the results of counts and measurements of the material referring to the first term (July 17).

Table 1

| No. of plant | Number of leaves | Number of circles of laticiferous vessels | L:V | Thickness of root bark in mm | Relation (5:3) | Number of cells between circles of laticiferons vessels |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 13 | 8 | 1.6 | 1.328 | 0.166 | 6—7 |
| 2 | 15 | 8 | 1.9 | 1.826 | 0.228 | 8—9 |
| 3 | 18 | 11 | 1.6 | 1.826 | 0.166 | 6—7 |
| 4 | 22 | 9 | 2.4 | 1.328 | 0.147 | 7—8 |
| 5 | 23 | 9 | 2.6 | 0.996 | 0.110 | 6—7 |
| 6 | 26 | 9 | 2.9 | 1.494 | 0.166 | 7—8 |
| 7 | 29 | 8 | 3.6 | 2.158 | 0.270 | 8—9 |
| 8 | 31 | 8 | 3.9 | 1.494 | 0.187 | 7—8 |
| 9 | 39 | 8 | 4.9 | 1.660 | 0.208 | 7—8 |
| 10 | 45 | 13 | 3.5 | 2.324 | 0.179 | 8—9 |
| 11 | 75 | 10 | 7.5 | 1.660 | 0.166 | 6—8 |

As may be seen from Table 1, along with a fairly stable rhythm observed in the formation of circles of laticiferous vessels (a sufficiently constant number of cells separating the circles), there is no such agreement between the number of circles of laticiferous vessels and that of leaves: the relation L : V fluctuates between 1.6 and 7.5. Noteworthy is the growth of the respective figures along with the increase in the number of leaves.

Let us analyse the data obtained from the second sampling (9th of September):

Table 2*

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 1 | 19 | 9 | 2.1 | 1.750 | 0.193 | 5—6 |
| 2 | 27 | 12 | 2.3 | 1.837 | 0.153 | 6—7 |
| 3 | 35 | 15 | 2.3 | 2.275 | 0.152 | 7—8 |
| 4 | 36 | 14 | 2.6 | 3.325 | 0.237 | 8—9 |
| 5 | 37 | 13 | 2.8 | 2.625 | 0.202 | 7—8 |
| 6 | 53 | 15 | 3.6 | 3.150 | 0.210 | 7—8 |
| 7 | 69 | 15 | 4.6 | 3.325 | 0.221 | 7—8 |
| 8 | 71 | 15 | 4.7 | 3.500 | 0.233 | 6—7 |
| 9 | 88 | 17 | 5.2 | 3.587 | 0.211 | 7—8 |
| 10 | 89 | 15 | 5.9 | 3.237 | 0.216 | 6—8 |
| 11 | 112 | 20 | 5.6 | 3.052 | 9.153 | 5—7 |
| 12 | 128 | 19 | 6.7 | 3.675 | 0.196 | 6—7 |

* Headings in Table 2, as well as in all that follow, are the same as in Table 1.

Similar to the former case, we find again a varying relation between L and V, which goes once more to confirm the absence of any direct relation between the cyclicity of leaf-formation and the rhythm of the formation of circles of laticiferous vessels. This time the total number of leaves per plant, as well as the number of circles of laticiferous vessels in the roots and the thickness of the root bark is larger than with the first sampling. In contrast to the first sampling, the figures of Table 2 bring out with sufficient clarity a direct connexion between the number of leaves and the thickness of the root bark, on the one hand, and between the thickness of the bark and the number of circles of laticiferous vessels, on the other.

The relation L : V, as derived from the second sampling, is found to fluctuate within a narrower range, between 2.1 and 6.7 as against 1.6 and 7.5 we had for the first sampling. This points to a closer relation between the number of circles of laticiferous vessels and that of leaves.

The analysis of the material gives us ground for interpreting this relation in the following way. The slow formation of the rosette and the relatively rapid growth of the rootlets at the earliest developmental stages is a well-known fact. Thus, at early stages of development the rapid increase in the number of circles of laticiferous vessels almost coincides with the increase in the number of leaves (which is borne out by the data referring to the small backward plants with few leaves from the first sampling). This is why in the most backward specimens the relation L : V is a particularly low one (1.6—2.0).

At later stages the formation of the rosette proceeds more rapidly, as well as the increase of leaf number per rosette (which is expressed in more successful specimens of the first sampling). The rate of differentiation of the circles of laticiferous vessels, which stands in no direct relation to the course of formation of leaves, still remains a relatively slow one. Under these conditions a rapid increase in the number of leaves, the rate of the formation of the circles of laticiferous vessels already beginning to slow down, leads to a considerable increase of the relation L : V. Now the rosettes which have completed their development begin intensely to supply the roots with assimilates, thus securing an enhanced growth of the former. The more leaves were formed at the early stage of growth, the more intensely proceeds the thickening of the root. Along with this, an enhanced growth of the root in the transverse direction, on account of the rhythm of the formation of circles of laticiferous vessels peculiar to it, is accompanied by a formation of a larger number of the latter.

Thus, at later stages those plants which have produced more leaves are likely to form the thickest roots and accordingly the largest number of circles of laticiferous vessels. In this case the relation between the characters: number of leaves and number of circles of laticiferous vessels, is a secondary rather than a primary one, and is due to a better growth made by those roots which sent out more leaves, the rate of differentiation of cells of the meristeme into laticiferous vessels being more or less constant.

In more advanced stages, when the growing and formative processes cease almost altogether, while the root has completed its development, largely according to the number of leaves, we were able to observe a still better agreement between the number of leaves, on the one hand, and the thickness of the bark and the number of circles of laticiferous vessels, on the other. At the same time the relation L : V is stabilized.

What has been said above is illustrated by the figures of Table 3, obtained from a treatment of samples taken on October 23. It should be noted that by this time some of the leaves of the rosettes already died off, and the total number of leaves in separate plants does not reach that recorded for the preceding sample. The indices for the thickness of roots and number of circles of laticiferous vessels proved in better agreement.

Similar relations have been observed for krym-saghyz, as well. A typical feature which distinguishes the latter from kok-saghyz consists in its being poorer in leaves, because of which the ratio L : V is lower.

Table 3

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 1 | 37 | 10 | 3.7 | 3.325 | 0.332 |
| 2 | 57 | 13 | 4.3 | 3.850 | 0.296 |
| 3 | 59 | 11 | 5.4 | 2.625 | 0.239 |
| 4 | 67 | 17 | 3.9 | 6.597 | 0.388 |
| 5 | 69 | 13 | 5.3 | 3.500 | 0.270 |
| 6 | 71 | 13 | 5.5 | 4.637 | 0.357 |
| 7 | 74 | 13 | 5.7 | 3.937 | 0.303 |
| 8 | 74 | 15 | 4.9 | 3.500 | 0.233 |
| 9 | 81 | 20 | 4.05 | 5.600 | 0.280 |
| 10 | 82 | 20 | 4.1 | 4.812 | 0.241 |
| 11 | 83 | 18 | 5.2 | 5.075 | 0.282 |

This goes once more to confirm that the cyclity in leaf-formation is in no direct relation to the rhythm of formation of circles of laticiferous vessels, for the scheme of leaf-arrangement in kok-saghyz and in krym-saghyz is similar.

As a typical feature of kok-saghyz we should point out its being more liable to the formation of several rosettes: while in krym-saghyz the presence of 2—3 rosette is of rather rare occurrence, in kok-saghyz one cannot infrequently observe 3—5 rosettes and even more. It has, however, no specific influence upon the rhythm of formation of circles of laticiferous vessels, save for the fact that such plants as have several rosettes and, consequently, a larger number of leaves, use to produce thicker roots with more numerous circles of laticiferous vessels. As a rule, they show also a higher L : V ratio.

Conclusions. 1. The rhythm of formation of circles of laticiferous vessels does not depend directly on the cyclicity of leaf-formation (we understand here the successive completion of cycles of leaf-arrangement), its dependence on the intensity of leaf-formation being a secondary one: the more numerous the leaves produced within the period of intense leaf-formation, the better do they secure the subsequent thickening of the root and the formation of a larger number of circles of laticiferous vessels in it. The latter are formed in a definite and sufficiently strict sequence; the physiological regularities by which it is controlled stand in need for special investigation.

2. The correlations brought out by the present work show that a plant should be stimulated to produce at the earliest stage a maximum number of leaves, and later on to develop an intense thickening of the root. This will induce the formation of more numerous circles of laticiferous vessels, and thus to an increase in the plant's rubber content.

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*PLANT PHYSIOLOGY*

# INTERACTION BETWEEN ROOTS AND SOIL COLLOIDS AS ONE OF THE PROBLEMS OF THE PHYSIOLOGY OF MINERAL NUTRITION OF PLANTS

## III. AGE VARIATION IN THE DESORBING ABILITY OF THE PLANT. INFLUENCE OF WILTING IN THE CASE OF MOISTURE DEFICIENCY

By E. I. RATNER

*(Communicated by A. N. Bach, Member of the Academy, 5. VI. 1943)*

The activity of a plant in its interaction with soil colloids is strongly affected by the changes which take place in the colloidal-chemical and osmotic properties of its plasm, according as the plant as a whole and its separate parts grow older. In his previous note (1) the author shows that the plant's absorbing system suffers a marked decrease in activity along with ageing; the phenomenon was viewed at the angle of this system's capacity to prevent the cations contained in the plant from being desorbed by the soil colloids under adequate conditions (*viz.* if the plant is high in these cations, while the colloidal complex of the soil is poor in them). It was but natural to assume that the reverse process which is more important in point of agricultural practice, *viz.* the process of desorption by the roots of exchange cations from the colloidal complex of the soil, would also be influenced by the bulk of age variations referred to above.

In order to clear up this question, we carried out an experiment with oats of the variety Wiener 01163. The plants were grown in water cultures, under moderate supply of mineral elements, inclusive of potassium; this was made in order to obtain plants apt to show a sufficient activity in absorbing this cation from the external medium.

At first the plants were kept on a $^1/_4$ *n* Helriegel solution; as soon as they started tillering, on account of a considerable increment of the mass and of the exhaustion of the nutrient solution, the mixture was added a $^1/_2$ *n* of Helriegel's solution without renewing the solution.

During the phases of tillering and flowering, after the roots had been washed off the nutrient solution, the plants were transferred under frequent air-blowing for 48 hours to a KCl solution, or to a soil suspension saturated up to 50 per cent of its exchange capacity with potassium, and to the remaining 50 per cent, with calcium. The amount of potassium supplied to the plants in the form of KCl or in the exchange form was equal to 120 mg per vessel in either case.

We confined ourselves to the two developmental stages mentioned above, since the tests described in our preceding note (1) had brought out with particular clearness that it was just within the period of flowering that the plant suffers a sharp drop in its fixing capacity.

After having been left to interact for 48 hours with the source of potassium, the plants were analysed for their content of this element. In an analogous way

were also analysed several plants taken into analysis previous to the absorption experiment. The results are tabulated below (means from four replicates).

Table 1

Absorbed K in mg per Pot

| | Tillering | Flowering |
|---|---|---|
| From KCl solution | 24.65 ± 1.51 | 38.1 ± 1.88 |
| From soil suspension | 31.78 ± 0.13 | 11.2 ± 3.37 |

If the quantity of potassium taken up by the plants within the stage of tillering be put at 100, then its uptake during flowering will make up 154 per cent in the case of soluble potassium, whereas in the case of exchange potassium it will be equal to no more than 35 per cent. Now if we were to put at 100 the amount of potassium absorbed by the plants from the KCl solution, then its amount absorbed from an adsorbed fixed condition will make for plants at the stage of tillering 129 per cent, and for those at the stage of flowering, 29 per cent only. These figures go to illustrate with sufficient clearness the loss by the plant of its ability to desorb exchange soil cations, as it grows older.

We arrived at the same conclusion also in a different way, namely, by studying the variations in the plant's ability to absorb exchange potassium under conditions of a reduced saturation of the colloidal soil complex with this element, *i. e.* in the case of an increased stability of the bond of this cation and the colloidal complex of the soil. With this aim in view we carried out an experiment in which, along with vessels filled with a soil suspension saturated up to 50 per cent with potassium and to the remaining 50 per cent with calcium, we had also a set of vessels in which the plants were allowed the same quantity of potassium (120 mg per vessel) in the form of a suspension of the same soil saturated with potassium to 10 per cent only, while the rest fell to the share of calcium (90 per cent). As a result we obtained the following figures illustrating the absorption by plants of potassium from suspension.

Table 2

Absorbed K in mg per Vessel

| | Tillering | Flowering |
|---|---|---|
| From soil suspension saturated with K (50%) and Ca(50%) | 24.65±1.51 | 11.2 ± 3.37 |
| From soil suspension saturated with K (10%) and Ca(90%) | 15.09±1.43 | —0.3 ± 3.08 |

As may be seen from Table 2, a reduced saturation of the colloidal soil complex with potassium makes it much less accessible also to the young plants. In this case, however, the potassium uptake was as high as 61 per cent to the quantity absorbed from a soil complex high in potassium. Unlike this, with plants already at the stage of flowering, the reduction of potassium content from 50 per cent to 10 per cent of its exchange capacity made this potassium utterly inaccessible to the plants under the experimental conditions (short interaction between roots and soil suspension).

Summing up all that has been said above, as well as the data available on the age variations of the retaining ability of the plasm, we are able to state that the activity of the plant in its interaction with a colloidal soil complex decreases with age.

In so far as this character is related with changes that take place in the colloidal-chemical properties of a vegetable cell, the same effect is likely to be

brought about not only by age variations but also by various external agents capable to affect the plant's colloidal system in a similar way.

In order to verify this statement, we decided to study the influence of water deficiency upon the plant. In our choice of this environmental agent we were guided by the work of Maximov (2) who found that wilting of leaves of various plants is accompanied with a sharp increase in the permeability of their cells to electrolytes, which points to a change in the state of the colloids of the plasm if the latter is dehydrated.

The experiments were carried on along two different lines: we studied the variation in the fixing capacity of the plasm as affected by wilting, and the changes in the desorbing activity of the root in the absorption of the exchange potassium from the colloidal complex of the soil, taking place under the influence of the same agent.

In the first experimental series barley plants of the variety Wiener 01163 were grown in water cultures on Hellriegel's solution. When 3 weeks old, the plants of one of the sets of vessels were placed, directly after their roots had been washed off the nutrient solution, in a suspension of a soil saturated to 30 per cent of its exchange capacity with calcium, and to the remaining 70 per cent, with sodium. The plants of the other set were not transferred to soil suspension directly upon root-washing, but were subjected to preliminary wilting. To make them wilt the vessels of this set were evacuated of the liquid they contained, and the plants were placed again in the empty vessels in which they were kept during 20 hours (since 5 p. m. till 1 p. m. of the next day). As the atmosphere created within the vessels was saturated with moisture (the roots being humid), the withering of the plants proceeded at a low rate. However, towards the end of the 20-hour period the leaves entirely lost their turgor.

After the period of wilting was completed, the plants of this series were also transferred to soil suspension along with those of the control series. The turgor was soon found to be completely reestablished. After a 36-hour-long interaction between the roots and the soil suspension, the plants were analysed for their potassium content, with the following results.

Table 3

Content of K in mg per Pot

| | In mg | In % to control |
|---|---|---|
| Previous to test (control) . . . . . | 97.4 | 100 |
| Upon interaction with soil suspension, without preliminary wilting | 78.7 | 81 |
| Upon interaction with soil suspension, with preliminary wilting . . | 52.8 | 54 |

The figures tabulated above give a clear idea of the decrease in the fixing capacity of the plasm, taking place under the influence of wilting.

The results of the other experiment whose aim it was to determine the variations in the root's desorbing activity as affected by wilting, made us arrive at a similar conclusion. This experiment was carried on with barleys of the same variety and also with plants taken into experiment when three weeks old. The plants were exposed to wilting at 11 a. m. and kept without water supply until 10 a. m. of the next day, which made up 23 hours. After this both the plants subjected to wilting and the controls were transferred to three kinds of potassium sources: 1) to a KCl solution; 2) to a suspension of a soil saturated with potassium to 50 per cent, and with calcium to the remaining 50; and 3) to a suspension of the same soil, saturated to 10 per cent with potassium, and to the remaining 90, with calcium. The quantity of potassium offered to the plants was in all cases 120 mg per vessel. The roots were allowed to interact with the potassium source during 48 hours, under frequent ventilation (Table 4).

Table 4

Uptake of K by Plants*

| | Without preliminary wilting | Upon preliminary wilting |
|---|---|---|
| From KCl solution | $\frac{18.79 \pm 0.66}{100}$ | $\frac{8.66 \pm 0.15}{100}$ |
| From a suspension of soil saturated with K to 50% and with Ca to 50% | $\frac{13.41 \pm 0.53}{71}$ | $\frac{4.24 \pm 0.35}{49}$ |
| From a suspension of soil saturated with K to 10% and with Ca to 90% | $\frac{9.34 \pm 0.45}{50}$ | $\frac{1.69 \pm 0.31}{19}$ |

* The numerator denotes the absolute amount of the potassium absorbed in mg per vessel; the denominator, the relative quantity, the amount absorbed from the KCl solution being put at 100.

It should be pointed out in the first line that the effect of wilting is sharply to decrease the plant's ability to absorb potassium even in a soluble form (Table 4). We have observed the same phenomenon with other plants (kenaph) in the case of the absorption of chlorine ions: a short wilting of the plants, which did not affect the ultimate yield, materially resulted in the subsequent in a considerable decrease of their capacity to absorb chlorine ions from the soil, under conditions of liberal water-supply.

The above-said may be illustrated by the following figures referring to the content of chlorine in the leaves and stalks of $1^1/_2$-month-old kenaph plants, 9 days upon the addition to the soil of a NaCl solution at a ratio of 2 equiv. per 100 g of the soil (Table 5). Previous to the addition of the NaCl solution, the plants subjected to wilting were withheld from watering during several days, owing to which the plants began to wither, and by the end of the wilting period their leaves showed for two days a total loss of turgor. Along with the supply of the NaCl solution, the soil in the vessels with those plants which had been subjected to wilting, was brought to a normal humidity (70 per cent to total moisture content).

Table 5

Content of Cl in per cent

| | In leaves | | In stalks | |
|---|---|---|---|---|
| | without salt (control) | with salt | without salt (control) | with salt |
| Plants not exposed to wilting | 0.72 | 1.41 | 0.70 | 1.88 |
| Plants exposed to wilting | 0.70 | 1.10 | 0.69 | 1.66 |

Coming back once more to the experiments with barley, it should be noted that the figures tabulated bring out a sharp drop in the plant's ability to desorb the exchange potassium from the absorbing soil complex; this drop is especially pronounced in the case where the degree of saturation of the complex with potassium is reduced, and the potassium is therefore more stably retained by the soil.

Summing up our data on the influence of wilting upon the variation in the activity of the plant's colloidal system in its interaction with the colloids of the soil, it should be pointed out that these changes proceed in the same direction as those connected with age, being evidently due to causes of a similar nature.

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*PHYSIOLOGY*

# HYPOCAPNIC AND ANOXAEMIC EFFECT ACCOMPANYING VARIATION OF THRESHOLD VALUES OF COLOUR PURITY

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The object of the present work has been to investigate the functional disturbances in colour perception that accompany the ascent of man to altitude. Reduced colour sensitivity under condition of low barometric and partial gas pressure was reported by several authors ([1–5]); it is more sharply pronounced in the case of blue and green, and less expressed with the red.

The thresholds values of colour purity were measured by us by way of comparing the chromatic field with an achromatic one of an equal luminosity. Comparison was made after the method of mixing on a disk of Maxwell, adopted from a work by N. T. and V. I. Fedorov ([3]).

Four coloured disks were used for the tests, a red, a yellow, a green and a blue one; they have been so assorted according to Mensell's atlas as to be of about the same luminosity and saturation; the red one was so chosen as to be complementary to the green one, while the yellow was complementary to the blue.

The colorimetrical characteristics of the disks was as follows:

| | | | | |
|---|---|---|---|---|
| 1. Red | (complem.) | $\lambda_g = 498$ mμ* | $p = 0.103$ | $L = 0.55$ |
| 2. Yellow | » | $\lambda = 576$ mμ | $p = 0.123$ | $L = 0.50$ |
| 3. Green | » | $\lambda = 521$ mμ | $p = 0.106$ | $L = 0.53$ |
| 4. Blue | » | $\lambda = 487$ mμ | $p = 0.072$ | $L = 0.62$ |

* The spectrum containing no purple colour, the hue of the latter is characterized by a reference to the wave-length of the spectrum colour complementary to the given purple colour.

where $\lambda$ is the prevailing wave-length; $p$, the relative saturation; and $L$, the reflection factor.

The coefficient of reflection of the grey disk was 0.54.

The object of observation was a rotating disk, the internal field of which, 55 mm in diameter, was achromatic, while the external one, 90 mm in diameter, was of one or another colour saturation. The observer was placed at a distance of 1 meter from the disk. The brightness of the field to be measured and of the adaptation field was equal to 23.2 foot-candles.

All the tests were carried out in a low pressure chamber illuminated with a 200 W daylight lamp.

Method. The observer was first, during 15 minutes, adapted to a given brightness, after which we proceeded to the measurement of the thresholds values of colour purity. The thresholds were determined by the angle of exposure of the coloured disk in degrees, at which the observer safely was found able to perceive the colour. The observers were presented the coloured disks in no regular order.

Results of Experiments. The tests involved four observers. Along with the basic sets, there were also carried out control experiments in a low pressure chamber, at an altitude of 300 m, in order to keep the experimental conditions constant (ventilation, noise of the motor).

The results of the control tests are given in Fig. 1; along the axis of abscissae is plotted the time in minutes, along the axis of ordinates are plotted the relative values of the threshold values of colour purity, expressed in degrees of exposure of the coloured disks. The large-scale curves refer to one of the four observers, the small-scale ones reproduce the arithmetical means derived from the results of all the tests with the four observers, the colours being the same. We found it allowable to average the data obtained with the four observers because of the analogous regularities revealed for all of them.

In Fig. 1 the extreme fluctuations of separate experimental points lie within the limits of 3°, whereas the averaged results show a scattering of the points within no more than one degree.

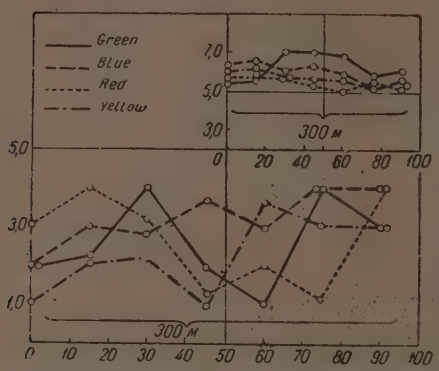


Fig. 1. Curves of threshold values of colour purity under conditions of normal atmospheric pressure

At the first stage of our work we determined the variation which took place in the threshold values of colour purity under conditions of anoxaemia, at a low barometric pressure. The experimental conditions were the following: the observer was lifted to a height of 300 m, at a rate of 10 m per second, with a platform; at this height at the beginning of the test and by the end of it two control points were determined. After this the observer was lifted to 5000 m and kept at this altitude until pronounced symptoms of anoxaemia appeared: dullness, drowsiness, slight cyanosis, which occurred approximately by the 60th on 70th minute of the experiment. The observer was then let down to 300 m. The determination of the thresholds was made every 10—15 minutes.

The results obtained are plotted in Fig. 2. We observe a clear-cut opposite run in the relation between the thresholds and the altitudinal factors. Whereas the sensitivity to the green and blue colour is reduced 2.5 times, the sensitivity to the red and the yellow increases as many times. It should be noted that the reduction in the sensitivity to green and blue was recorded in all tests involving the raising of the observer to some altitude; in some of the tests it reached 400—450 per cent. The increase of sensitivity to red and yellow was less stable: there was some discrepancy between the results of individual measurements made at an altitude, which may be seen in the averaged curves.

The data obtained might be interpreted as being due to anoxaemia. This suggested us the idea of removing the anoxaemic effect by means of oxygen.

Under similar experimental conditions, but for an oxygen respirator, the observer was lifted to an altitude of 6000 m. The results of this set of experiments are illustrated by Fig. 3. The results of the action of altitude upon the thresholds values of colour purity show the same course as under conditions of anoxaemia; however, absolute values of the thresholds are somewhat smaller than in the case where the ascent was effected under conditions of oxygen deficiency.

The third experimental set was carried out with the purpose of investigating the effect of rarefied atmosphere, our subsequent work being carried on with gas mixtures, at a low barometric pressure.

Tests involving inhalation of gas mixture at a lowered partial oxygen pressure gave results similar to those obtained with the rise in a variable pressure chamber. This fact made us conclude that the variation in the threshold

values of colour purity does not depend on alterations of barometric pressure.

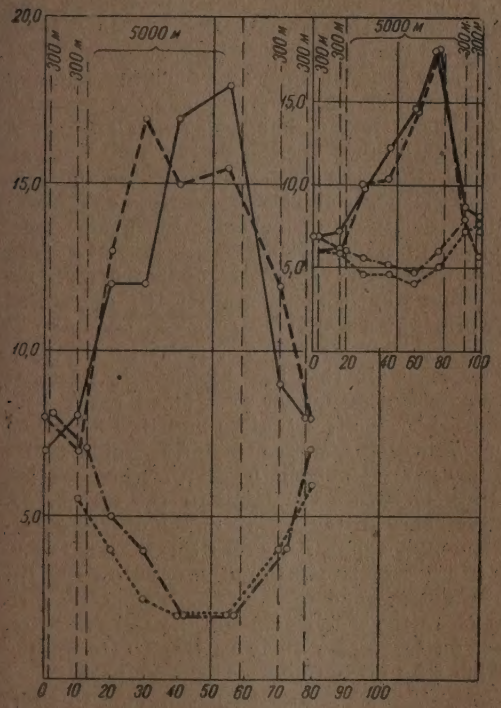


Fig. 2. Threshold values of colour purity as dependent on altitude*

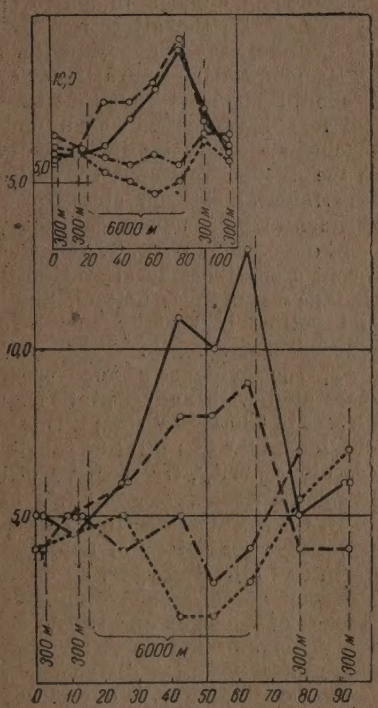


Fig. 3. Curves of threshold values of colour purity under conditions of reduced barometric pressure (observer given oxygen to inhale)

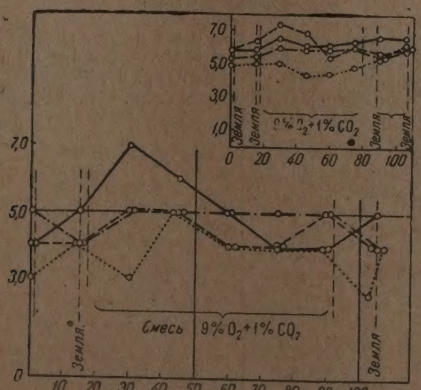


Fig. 4. Curves of threshold values of colour purity obtained with observers given a gas mixture poor in oxygen ($O_2 = 9$ per cent), and oversaturated with carbon dioxide ($CO_2 = 1$ per cent)

* Denotation for Figs. 2, 3 and 4 is the same as in Fig. 1.

The low content of carbon dioxide in the alveolar air was reported by a number of workers who were engaged in the study of the influence of altitudinal agents upon the organism. The physiological rôle of this phenomenon remains, however, obscure and subject to controversy. In order to investigate the part played by hypocapnia in the alterations of colour perception revealed by us, we carried out one more set of experiments. This time the gas mixture which corresponded by its composition to an altitude of 5500 m, was added 1 per cent of carbon dioxide. It should be pointed out that the addition of carbon dioxide made the observer feel less uncomfortable than under the conditions of anoxaemia; its effect was to change sharply the influence of anoxaemia upon the thresholds. The variation in the latter was now so insignificant that the curves obtained in the last experimental set run close to those of the controls, the scattering of the experimental points being almost within the limits of measurement errors.

Conclusions. 1. The authors have found under the conditions of a rapid ascent a considerable alteration in the threshold values of colour purity. Unlike the rest of investigators, they have found out that the sensitivity to the red and yellow is thereby increased, while the sensitivity to green and blue is reduced considerably.

2. The evidence secured permits to ascertain that the hypocapnia which is associated with anoxaemia, is an agent affecting colour perception under conditions of photopic vision; this is evidenced by the fact that an addition of carbon dioxide to the gas mixture inhaled prevents disturbances of the threshold values of colour purity, irrespective of the low partial oxygen pressure. When inhaled at some altitude, oxygen does but slightly reduce the shifts in the thresholds.

3. Great interest belongs to the divergence in the course of the curves obtained under conditions of anoxaemia and hypocapnia: whereas the sensitivity to green and blue decreases, an increase in the sensitivity to red and yellow sets in. Different colour-sensitive apparatus do not respond in the same way to the influence of the same agent. The variation in the sensitivity to blue and green, on the one hand, and to yellow and red, on the other, make us think of a unique cause underlying the disturbances which take place in either of these pairs.

In concluding it is our pleasant duty to express our deep gratitude to Prof. M. P. Brestkin for many a valuable advice in the course of this work.

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